Observer-Based Fully Distributed Containment Control for MASs Subject to DoS Attacks

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***Abstract*—The problem of the observer-based fully distributed containment control for multiagent systems (MASs) subject to denial-of-service (DoS) attacks is investigated in this article. First, a switched fully distributed control framework is established for a class of DoS attacks constrained by the attack duration. Then, a novel attack-resilient control scheme is developed to accomplish the containment control task. The major advantages of the devised control scheme are that any information of the whole network topology structure is not involved and only the information from neighbor agents is used. What is more, a novel observer-based attack compensator is devised to resist DoS attacks. Finally, a practical example of the mobile robot system is presented to testify the validity of the designed control scheme by a comparison.**

***Index Terms*—Attack compensation, containment control, multiagent systems (MASs), observer-based fully distributed control.**

I. Introduction

THE COOPERATIVE control problems have attracted much attention owing to their convenience, energy saving, and high efficiency in recent years [1]–[3]. As one of the methods of the cooperative control, containment control has received great research and attention. Agents are divided into leaders and followers in the containment control problem, and its control task is to enable some followers to reach on the convex hull composed of multiple leaders. Up to now, a variety of containment control problems have been studied. The finite-horizon H_∞ containment control problem is addressed in [4]. The fault-tolerant fuzzy containment control scheme is devised in [5] to accomplish the states containment. Moreover, the sampled-based formation containment control problem is addressed in [6].

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It should be pointed out that the smallest or largest eigenvalue of the Laplacian matrix is used in the control algorithm in the above researches. However, for each agent, the smallest or largest eigenvalue is the global information due to the agent needs to obtain the whole network topology structure to calculate it. Considering that agents are not always able to obtain the knowledge of the whole network topology structure, a fully distributed control scheme is devised in [7]–[12], in which the global information is not involved. Therefore, compared with the traditional distributed control scheme, the major strong point of the fully distributed control is that it only uses the local information of neighbors and its own to achieve the control task. In [7] and [8], the fully distributed consensus control problems under undirected and directed graphs are studied, respectively. The fully distributed consensus control problems with the event-triggered communication scheduling are addressed in [9] and [10]. In [13], the fully distributed containment controller devise problem of multiagent systems (MASs) suffered from disturbances is investigated. However, the actual system state is used in the control scheme in [13], which is usually not available as mentioned in [14] and [15]. Therefore, the observer-based control scheme is more practical. In [16], the observer-based fully distributed consensus control problem is studied. The problem of the cooperative output regulation is addressed by devising the event-triggered fully distributed observer in [17]. In addition, the problem of the fully distributed fault-tolerant synchronization with an adaptive observer is addressed in [18]. It is noted that the observer designed in [16]–[18] is in a distributed form, which needs to use the system information transmitted from neighbor agents. However, transmitting these information requires additional communication resources. Consequently, for the purpose of saving communication resources, it is a better idea to design observer based on its own system information, which is the focus of this article.

Since the use of communication networks and the emergence of network attacks, the security control problems of MASs are widely concerned. It should be pointed out that the malicious network attacker can greatly affect the system [19]–[23]. Consequently, it is significant to study the security control problems under cyber attacks. There are commonly two types of cyber attacks: 1) denial-of-service (DoS) attacks and 2) false data injection (FDI) attacks, in which FDI attacks aim to inject the error vectors into the system and DoS attacks aim to block the information transmission channel such that the useful information cannot be transmitted to the controller. So far, many security control problems under the influence of network attacks have been studied in [24]–[34].

In [25], the containment control problem for MASs under FDI attacks is addressed. The output formation containment control scheme for heterogeneous MASs with DoS attacks is devised in [26]. The containment control problem under DoS attacks is addressed in [27]. Unfortunately, up to now, there are few researches on the fully distributed security control. In [29], the fully distributed consensus security control problem subject to DoS attacks is addressed, in which the dynamic of each agent is considered to be first order. Consequently, it is significant to study the fully distributed security control problem for general high-order systems under the influence of DoS attacks.

Notice that the attack compensation approach has not been designed in the related results mentioned above. For the purpose of mitigating the influence of network attacks, the attack compensation approach is developed in [35]–[38]. In [35] and [36], the observer-based control problems with the static and the dynamic event-triggered mechanism are investigated under DoS attacks, respectively, in which a switched observer is devised to mitigate the influence of DoS attacks. But the impact of DoS attacks on the controller is not addressed, that is, the control input equals zero when the communication networks suffer from DoS attacks. The security control problem is studied in [37], where a novel compensation approach is developed to mitigate the influence of DoS attacks. Specifically, the historical system information is utilized to compensate for the lost information during the attack. Moreover, for the purpose of further mitigating the influence of DoS attacks, Yang *et al.* [38] designed an open-loop observer to estimate the system information during DoS attacks based on the latest successfully received system information. However, notice that the control scheme designed in [35]–[38] is not in a fully distributed form and uses the global information of the network topology. Consequently, how to design a fully distributed control scheme with an attack compensation approach to mitigate the influence DoS attacks is a meaningful problem to be studied.

Inspired by the above-mentioned problems, the fully distributed containment control problem of MASs subject to DoS attacks with a novel attack compensation approach is studied, which has the following contributions.

1) Compared with the existing distributed control results against DoS attacks [35]–[38] in which the global network topology information is used, a novel control framework in a fully distributed manner under DoS attacks is established in this article. Based on which an attack-resilient fully distributed control scheme is designed to accomplish the containment control task. Besides, different from the fully distributed security control problem studied in [29] for first-order MASs, the general high-order MASs are studied in this article.
2) Unlike [16]–[18] that a distributed observer using the neighbor information is designed to accomplish the fully distributed control task, the observer designed in this article for each agent only uses its own system information, which saves the limited communication resources.
3) An active attack compensation scheme is developed to further alleviate the effect of DoS attacks on the system performance, in which an open-loop observer is devised to reconstruct the attacked system states of neighbor agents based on the latest successfully received system state.

The remainder of this article is organized as follows. The fully distributed containment control framework formulation with DoS attacks is presented in Section II. In Section III, the main results of this article are provided. A mobile robot system is presented to testify the validity of the designed control scheme in Section IV. Our conclusions are summarized in Section V.

Notations: $\otimes$ stands for the Kronecker product. $\lambda_{\max}(X)$ stands for the largest eigenvalue of X and $\lambda_{\min}(X)$ stands for the smallest eigenvalue. In the matrix, $*$ denotes the symmetry term. $\|\cdot\|$ stands for the 2-norm. $M \backslash N$ represents a set belongs to M but not N. I_M stands for the M-dimensional identity matrix. diag$\{\cdot\}$ stands for the diagonal matrix. $Y > 0$ indicates the positive-definite matrix. $\mathbb{R}^{n\times n}$ denotes the set of all $n \times n$ real matrices. $\mathbb{R}^n$ denotes the n-dimensional real number space. $\mathbb{N}$ denotes the positive real number. $x(t^-)$ represents the left-hand limits of x at t. $\bar{a}$ and $\underline{a}$ represent the upper and lower bounds of a, respectively.

II. Preliminaries and Problem Formulation

A. Preliminaries

The network topology of MASs is defined as a graphs $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, in which $\mathcal{V} = \{1, \ldots, N\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represent the set of nodes and edges. Define the adjacency matrix as $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$, in which $a_{ij} = 1$ and $a_{ii} = 0$ when $(i, j) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. The neighborhood of the ith agent is represented as $N_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}\}$. Besides, the self-edge (i, i) is not allowed. The Laplacian matrix $L = D - \mathcal{A}$ with $D = \text{diag}\{d_1, \ldots, d_N\}$ and $d_i = \sum_{j=1, j\neq i}^{N} a_{ij}$.

It is supposed that MASs have M followers and $N - M$ leaders, which are defined as $\mathcal{F} \triangleq \{1, \ldots, M\}$ and $\Omega \triangleq \{M + 1, \ldots, N\}$. Considering leader does not receive any information from neighbor agents, we have the following Laplacian matrix:

$$L = \begin{bmatrix} L_1 & L_2 \\ 0_{(N-M)\times M} & 0_{(N-M)\times(N-M)} \end{bmatrix}$$

with $L_1 \in \mathbb{R}^{M\times M}$ and $L_2 \in \mathbb{R}^{M\times(N-M)}$ being the interaction of followers and the connection between leaders and followers, respectively.

Assumption 1: For each follower, there is at least one connection to the leader.

Remark 1: Assumption 1 is reasonable since if there is a follower that cannot receive any information from at least one leader, the containment control task will fail for this follower due to it loses the reference trajectory.

Lemma 1 [4]: Under Assumption 1 and undirected graph, $L_1 > 0$. The sum of each row of $-L_1^{-1}L_2$ is one and its each element is non-negative. Then, there exists matrix S so that

$$S^T L_1 S = \Lambda$$

where $\Lambda = \text{diag}\{\lambda_1, \ldots, \lambda_M\}$ with λ_i being the eigenvalue of L_1.

B. Fully Distributed Controller Design

The ith subsystem of followers is as follows:

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t) \\ y_i(t) = Cx_i(t), i \in \mathcal{F} \end{cases} \quad (1)$$

where $x_i(t) \in \mathbb{R}^n$ is the system state. $u_i(t) \in \mathbb{R}^m$ is the control input. $y_i(t) \in \mathbb{R}^d$ is the system output. A, B, and C are constant matrices, and assume that (A, C) and (A, B) are observable and controllable, respectively.

The lth subsystem of leaders is considered as follows:

$$\dot{x}_l(t) = Ax_l(t), l \in \Omega. \quad (2)$$

Due to the fact that not all states of MASs are available in the actual situation, we devise the following observer to estimate them:

$$\begin{cases} \dot{\hat{x}}_i(t) = A\hat{x}_i(t) + Bu_i(t) + G\big(y_i(t) - \hat{y}_i(t)\big) \\ \hat{y}_i(t) = C\hat{x}_i(t), i \in \mathcal{F} \end{cases} \quad (3)$$

where $\hat{x}_i(t) \in \mathbb{R}^n$ is the estimated system state. $\hat{y}_i(t) \in \mathbb{R}^d$ is the estimated system output. $G \in \mathbb{R}^{n\times d}$ is the observer gain.

To accomplish the fully distributed containment control task, the following controller is devised:

$$\begin{aligned} u_i(t) &= \alpha_i(t)K\hat{\delta}_i(t) \\ \dot{\alpha}_i(t) &= \text{Proj}_{[\underline{\alpha}_i,\bar{\alpha}_i]}\{\alpha_i(t)\} \\ &= \begin{cases} 0, & \text{if } \alpha_i(t) = \bar{\alpha}_i \text{ and } \Phi_i > 0 \\ & \text{or } \alpha_i(t) = \underline{\alpha}_i \text{ and } \Phi_i < 0 \\ \kappa_i\Phi_i, & \text{otherwise} \end{cases} \end{aligned} \quad (4)$$

where $\Gamma \in \mathbb{R}^{n\times n}$ and $K \in \mathbb{R}^{m\times n}$ are the feedback gain matrices. $\Phi_i = -\beta_i\alpha_i(t) + \hat{\delta}_i^T(t)\Gamma\hat{\delta}_i(t)$. $\hat{\delta}_i(t) = \sum_{j=1}^{M} a_{ij}(\hat{x}_i(t) - \hat{x}_j(t)) + \sum_{l=M+1}^{N} a_{il}(\hat{x}_i(t) - x_l(t))$ is the estimated local neighbor error. κ_i and β_i are positive constants to be reasonably selected.

Remark 2: Proj$\{\cdot\}$ stands for the projection operator, which has been used in the fully distributed controller design [39], [40]. Due to the emergence of network attacks, the existing fully distributed control schemes and stability analysis methods presented in [7]–[10] will no longer be applicable. Therefore, we introduce the projection operator into the control scheme to restrict the adaptive parameters $\alpha_i(t)$ to the interval $[\underline{\alpha}_i, \bar{\alpha}_i]$, which facilitates the subsequent stability analysis.

Then, combining with (1), (3), and (4), we can get

$$\begin{aligned} \dot{x}_f(t) &= (I_M \otimes A)x_f(t) + (\Psi(t) \otimes BK)\hat{\delta}(t) \\ \dot{\hat{x}}_f(t) &= (I_M \otimes A)\hat{x}_f(t) + (\Psi(t) \otimes BK)\hat{\delta}(t) \\ &\quad + (I_M \otimes GC)(x_f(t) - \hat{x}_f(t)) \end{aligned} \quad (5)$$

with

$$\begin{aligned} \hat{x}_f(t) &= \left[\hat{x}_1^T(t), \ldots, \hat{x}_M^T(t)\right]^T \\ x_f(t) &= \left[x_1^T(t), \ldots, x_M^T(t)\right]^T \\ \Psi(t) &= \text{diag}\{\alpha_1(t), \ldots, \alpha_M(t)\} \\ \hat{\delta}(t) &= \left[\hat{\delta}_1^T(t), \ldots, \hat{\delta}_M^T(t)\right]^T. \end{aligned}$$

According to (2), we can get the following augmented system of leaders:

$$\dot{x}_L(t) = (I_{N-M} \otimes A)x_L(t) \quad (6)$$

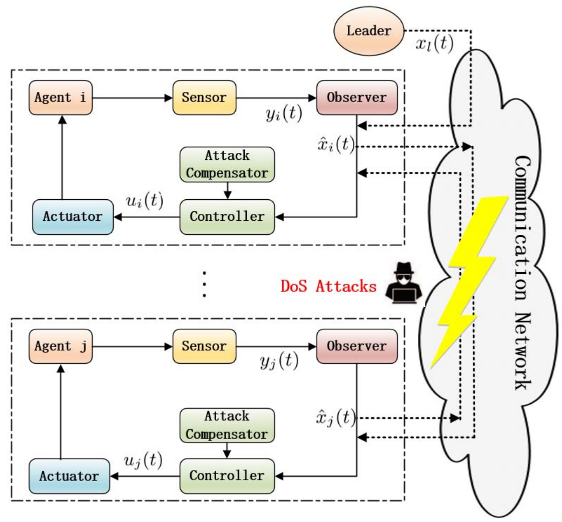

Fig. 1. System framework with DoS attacks.

where

$$x_L(t) = \left[x_{M+1}^T(t), \ldots, x_N^T(t)\right]^T.$$

Defining the estimation error of the follower as $\tilde{x}_i(t) = x_i(t) - \hat{x}_i(t)$, we have

$$\dot{\tilde{x}}_f(t) = (I_M \otimes (A - GC))\tilde{x}_f(t) \quad (7)$$

where

$$\tilde{x}_f(t) = \left[\tilde{x}_1^T(t), \ldots, \tilde{x}_M^T(t)\right]^T.$$

Defining the containment error as $e(t) = x_f(t) + (L_1^{-1}L_2 \otimes I_{N-M})x_L(t)$, we can get

$$\begin{aligned} \dot{e}(t) &= (I_M \otimes A)x_f(t) + (\Psi(t) \otimes BK)\hat{\delta}(t) \\ &\quad + \left(L_1^{-1}L_2 \otimes I_{N-M}\right)(I_{N-M} \otimes A)x_L(t) \\ &= (I_M \otimes A)e(t) + (\Psi(t) \otimes BK)\hat{\delta}(t). \end{aligned} \quad (8)$$

C. DoS Attacks Modeling and Problem Presentation

The system diagram with DoS attacks is presented in Fig. 1, in which DoS attacks intend to destroy the communication networks such that the information transmission between agents will be interrupted. Once the agent cannot receive the information sent by all its neighbors, it will not be able to generate the normal control input and the control task will not be completed.

Assume that the nth DoS attack interval is $[T_n^{\text{on}}, T_n^{\text{off}})$ with T_n^{on} and T_n^{off} representing the start time and the end time of the nth DoS attack. The total duration with DoS attacks in $[0, t]$ is given by

$$\Xi_a(0, t) = \left\{\bigcup_{n\in\mathbb{N}} [T_n^{\text{on}}, T_n^{\text{off}})\right\} \bigcap [0, t]. \quad (9)$$

Correspondingly, we have

$$\Xi_s(0, t) = [0, t] \setminus \Xi_a(0, t). \quad (10)$$

For the purpose of mitigating the effect of DoS attacks, inspired by [38], we devise the following controller with an attack compensator:

$$\begin{aligned} u_i(t) &= \alpha_i(t)K\tilde{\delta}_i(t) \\ \dot{\alpha}_i(t) &= \mathrm{Proj}_{[\underline{\alpha}_i,\bar{\alpha}_i]}\{\alpha_i(t)\} \\ &= \begin{cases} 0, & \text{if } \alpha_i(t)=\bar{\alpha}_i \text{ and } \tilde{\Phi}_i>0 \\ & \text{or } \alpha_i(t)=\underline{\alpha}_i \text{ and } \tilde{\Phi}_i<0 \\ \kappa_i\tilde{\Phi}_i, & \text{otherwise} \end{cases} \end{aligned} \tag{11}$$

where $\tilde{\delta}_i(t) = \sum_{j=1}^{M} a_{ij}(\hat{x}_i(t)-\tilde{x}_j(t))+\sum_{l=M+1}^{N} a_{il}(\hat{x}_i(t)-\tilde{x}_l(t))$, $\tilde{\Phi}_i = -\beta_i\alpha_i(t) + \tilde{\delta}_i^T(t)\Gamma\tilde{\delta}_i(t)$ and the attack compensator is devised as follows:

$$\begin{cases} \dot{\tilde{x}}_j(t) = A\tilde{x}_j(t),\ t\in\left[T_n^{\mathrm{on}}, T_n^{\mathrm{off}}\right) \\ \tilde{x}_j(t) = \hat{x}_j\left(t^-\right),\ \ t=T_n^{\mathrm{on}}, j\in\mathcal{F} \\ \dot{\tilde{x}}_l(t) = A\tilde{x}_l(t),\ t\in\left[T_n^{\mathrm{on}}, T_n^{\mathrm{off}}\right) \\ \tilde{x}_l(t) = x_l\left(t^-\right),\ \ t=T_n^{\mathrm{on}}, l\in\Omega. \end{cases}$$

Remark 3: It can be observed that when the communication networks suffer from DoS attacks, the system states sent from neighbor agents cannot be received by the corresponding agent. Then, for the purpose of mitigating the effect of DoS attacks, an open-loop observer is used as an attack compensator to restructure the system states of neighbor agents based on the latest successfully received ones.

Remark 4: Inspired by [38], a novel attack compensation approach based on the open-loop observer is designed. Considering that the system's own states are available and only the neighbor states are not available, the open-loop observer is just devised to estimate the neighbor information, which is different from [38] that the observer is devised to estimate all system states, including the states of its own and neighbor agents.

Then, we have the following switched dynamics of the containment error:

$$\begin{cases} \dot{e}(t) = (I_M\otimes A)e(t) + (\Psi(t)\otimes BK)\hat{\delta}(t),\ \ t\in\left[T_{n-1}^{\mathrm{off}}, T_n^{\mathrm{on}}\right) \\ \dot{e}(t) = (I_M\otimes A)e(t) + (\Psi(t)\otimes BK)\tilde{\delta}(t),\ \ t\in\left[T_n^{\mathrm{on}}, T_n^{\mathrm{off}}\right). \end{cases}$$

Taking into account the energy limitation of DoS attacks, the following assumption is presented to constrain the duration of DoS attacks.

Assumption 2: Letting $|\Xi_a(0,t)|$ be the total duration with DoS attacks in $[0,t]$, then we assume

$$|\Xi_a(0,t)| \le \Xi_0 + \frac{t}{\tau_a} \tag{12}$$

with $\tau_a > 1$ and $\Xi_0 > 0$ being constants.

Remark 5: Assumption 2 means that the duration with DoS attacks is restricted, which has been used in [25] and [27]. It is worth noting that considering the limited attack resources and the self-repair mechanism of the system, the assumption about the duration of DoS attacks is reasonable in the actual situations.

Problem 1: For the MASs subject to DoS attacks, this article intends to devise an observer-based fully distributed containment control scheme to guarantee that the containment error $e(t)$ is uniformly bounded, i.e., $e(t)$ converges to $\{e(t)|\ \|e(t)\|\le\chi\}$ with $\chi>0$ being a constant.

III. Main Results

This part intends to discuss the security analysis of MASs. The main results are presented in the following theorem.

Theorem 1: For given positive constants θ_1, $\tilde{\theta}_1$, ξ_1, θ_2, $\tilde{\theta}_2$, ξ_2, θ_3, $\tilde{\theta}_3$, and η, if there exist positive constant τ_a and positive-definite matrices $Q, P\in\mathbb{R}^{n\times n}$ such that (13)–(16) hold, then Problem 1 can be solved

$$AP + PA^T - \eta BB^T + \left(\theta_1+\tilde{\theta}_1\right)P < 0 \tag{13}$$

$$QA + A^TQ - 2C^TC + (\theta_2+\tilde{\theta}_2)Q < 0 \tag{14}$$

$$AP + PA^T - (\theta_3-\tilde{\theta}_3)P < 0 \tag{15}$$

$$\frac{\xi_1+\xi_2}{\xi_1} - \tau_a < 0. \tag{16}$$

Moreover, the gain matrix of observer and controller can be obtained as $G = Q^{-1}C^T$ and $K = -B^TP^{-1}$.

Proof: The Lyapunov function is selected as $V(t) = e^T(t)(L_1\otimes P^{-1})e(t) + \rho\tilde{x}^T(t)(I_M\otimes Q)\tilde{x}(t) + \sum_{i=1}^{M}(1/\kappa_i)(\alpha_i(t)-\varpi)^2$ with $\varpi>0$ and $\rho>0$ being determined later.

Note that there exist two cases for the derivative of the adaptive parameter $\dot{\alpha}_i(t)$, i.e., if $\alpha_i(t)=\bar{\alpha}_i$ and $\Phi_i>0$ or $\alpha_i(t)=\underline{\alpha}_i$ and $\Phi_i<0$, then $\sum_{i=1}^{M}(2/\kappa_i)(\alpha_i(t)-\varpi)\dot{\alpha}_i(t)=0$, and otherwise $\sum_{i=1}^{M}(2/\kappa_i)(\alpha_i(t)-\varpi)\dot{\alpha}_i(t)\ne 0$. Moreover, only the latter case is discussed since it includes the former case. In the absence of DoS attacks, i.e., $t\in[T_{n-1}^{\mathrm{off}}, T_n^{\mathrm{on}})$, we can get

$$\begin{aligned} \dot{V}(t) = {} & 2e^T(t)(L_1\otimes P^{-1}A)e(t) \\ & + 2e^T(t)(L_1\Psi(t)\otimes P^{-1}BK)\hat{\delta}(t) \\ & + 2\rho\tilde{x}^T(t)(I_M\otimes Q(A-GC))\tilde{x}(t) \\ & + 2\sum_{i=1}^{M}(\alpha_i(t)-\varpi)\left(-\beta_i\alpha_i(t)+\hat{\delta}_i^T(t)\Gamma\hat{\delta}_i(t)\right). \end{aligned} \tag{17}$$

From the definition of $\hat{\delta}_i(t)$ defined in (4), we have

$$\begin{aligned} \hat{\delta}(t) &= (L_1\otimes I_M)\hat{x}_f(t) + (L_2\otimes I_{N-M})x_L(t) \\ &= (L_1\otimes I_M)x_f(t) + (L_2\otimes I_{N-M})x_L(t) \\ &\quad + (L_1\otimes I_M)\hat{x}_f(t) - (L_1\otimes I_M)x_f(t) \\ &= (L_1\otimes I_M)\left[x_f(t) + \left(L_1^{-1}L_2\otimes I_{N-M}\right)x_L(t)\right] \\ &\quad - (L_1\otimes I_M)\tilde{x}(t) \\ &= (L_1\otimes I_M)(e(t)-\tilde{x}(t)). \end{aligned}$$

Similarly, we get $\delta(t) = (L_1\otimes I_M)(\hat{e}(t)+\tilde{x}(t))$ with $\hat{e}(t) = \hat{x}_f(t) + (L_1^{-1}L_2\otimes I_{N-M})x_L(t)$. Besides, $\delta(t) = (L_1\otimes I_M)x_f(t) + (L_2\otimes I_{N-M})x_L(t) = (L_1\otimes I_M)[x_f(t) + (L_1^{-1}L_2\otimes I_{N-M})x_L(t)] = (L_1\otimes I_M)e(t)$ and $\hat{\delta}(t) = (L_1\otimes I_M)\hat{e}(t)$. Then, we can get

$$\begin{aligned} \dot{V}(t) = {} & 2e^T(t)(L_1\otimes P^{-1}A)e(t) \\ & + 2\hat{\delta}^T(t)\left(\Psi(t)\otimes P^{-1}BK\right)\hat{\delta}(t) \\ & + 2\tilde{x}^T(t)\left(L_1\Psi(t)\otimes P^{-1}BK\right)\hat{\delta}(t) \\ & + 2\rho\tilde{x}^T(t)(I_M\otimes Q(A-GC))\tilde{x}(t) \\ & + 2\sum_{i=1}^{M}\alpha_i(t)\hat{\delta}_i^T(t)\Gamma\hat{\delta}_i(t) - 2\varpi\sum_{i=1}^{M}\hat{\delta}_i^T(t)\Gamma\hat{\delta}_i(t) \\ & + 2\sum_{i=1}^{M}(\alpha_i(t)-\varpi)(-\beta_i\alpha_i(t)). \end{aligned} \tag{18}$$

Letting $\Gamma = P^{-1}BB^TP^{-1}$ and $K = -B^TP^{-1}$, we have

$$2\sum_{i=1}^{M}\alpha_i(t)\hat{\delta}_i^T(t)\Gamma\hat{\delta}_i(t) = 2\hat{\delta}^T(t)(\Psi(t)\otimes P^{-1}BB^TP^{-1})\hat{\delta}(t).$$

Then, (18) is rewritten as follows:

$$\begin{aligned}\dot{V}(t) = {} & 2e^T(t)(L_1\otimes P^{-1}A)e(t)\\ & -\ 2\tilde{x}^T(t)(L_1\Psi(t)L_1\otimes P^{-1}BB^TP^{-1})e(t)\\ & +\ 2\tilde{x}^T(t)(L_1\Psi(t)L_1\otimes P^{-1}BB^TP^{-1})\tilde{x}(t)\\ & +\ 2\rho\tilde{x}^T(t)(I_M\otimes Q(A-GC))\tilde{x}(t)\\ & -\ 2\varpi\sum_{i=1}^{M}\hat{\delta}_i^T(t)P^{-1}BB^TP^{-1}\hat{\delta}_i(t)\\ & +\ 2\sum_{i=1}^{M}(\alpha_i(t)-\varpi)(-\beta_i\alpha_i(t)).\end{aligned} \tag{19}$$

The following transformation is made for the fifth term of (19):

$$\begin{aligned}& -2\varpi\sum_{i=1}^{M}\hat{\delta}_i^T(t)P^{-1}BB^TP^{-1}\hat{\delta}_i(t)\\ &= -2\varpi\hat{\delta}^T(t)(I_M\otimes P^{-1}BB^TP^{-1})\hat{\delta}(t)\\ &= -2\varpi\left[e^T(t)-\tilde{x}^T(t)\right]\left(L_1^2\otimes P^{-1}BB^TP^{-1}\right)\left[e(t)-\tilde{x}(t)\right]\\ &= -2\varpi e^T(t)\left(L_1^2\otimes P^{-1}BB^TP^{-1}\right)e(t)\\ &\quad -2\varpi\tilde{x}^T(t)\left(L_1^2\otimes P^{-1}BB^TP^{-1}\right)\tilde{x}(t)\\ &\quad +4\varpi e^T(t)\left(L_1^2\otimes P^{-1}BB^TP^{-1}\right)\tilde{x}(t)\\ &\le -2\varpi\left(1-\frac{1}{l}\right)e^T(t)\left(L_1^2\otimes P^{-1}BB^TP^{-1}\right)e(t)\\ &\quad -2\varpi(1-l)\tilde{x}^T(t)\left(L_1^2\otimes P^{-1}BB^TP^{-1}\right)\tilde{x}(t)\end{aligned}$$

where the inequality $2a^Tb \le a^Ta + b^Tb$ is used in the transformation of the fifth term of (19) and $l > 1$ is a constant.

Then, we can get

$$\begin{aligned}\dot{V}(t) < {} & 2e^T(t)(L_1\otimes P^{-1}A)e(t)\\ & -\ 2\tilde{x}^T(t)\left(L_1\Psi(t)L_1\otimes P^{-1}BB^TP^{-1}\right)e(t)\\ & +\ 2\tilde{x}^T(t)\left(L_1\Psi(t)L_1\otimes P^{-1}BB^TP^{-1}\right)\tilde{x}(t)\\ & +\ 2\rho\tilde{x}^T(t)(I_M\otimes Q(A-GC))\tilde{x}(t)\\ & -\ 2\varpi\left(1-\frac{1}{l}\right)e^T(t)\left(L_1^2\otimes P^{-1}BB^TP^{-1}\right)e(t)\\ & -\ 2\varpi(1-l)\tilde{x}^T(t)\left(L_1^2\otimes P^{-1}BB^TP^{-1}\right)\tilde{x}(t)\\ & +\ 2\sum_{i=1}^{M}(\alpha_i(t)-\varpi)(-\beta_i\alpha_i(t)).\end{aligned} \tag{20}$$

Letting $\bar{e}(t) = (S^T\otimes P)e(t)$ leads to

$$\begin{aligned}\dot{V}(t) < {} & 2\bar{e}^T(\Lambda\otimes AP)\bar{e}(t)\\ & -\ 2\tilde{x}^T(t)\left(L_1\Psi(t)L_1\otimes P^{-1}BB^TP^{-1}\right)e(t)\\ & +\ 2\tilde{x}^T(t)\left(L_1\Psi(t)L_1\otimes P^{-1}BB^TP^{-1}\right)\tilde{x}(t)\\ & +\ 2\rho\tilde{x}^T(t)(I_M\otimes Q(A-GC))\tilde{x}(t)\\ & -\ 2\varpi\left(1-\frac{1}{l}\right)\bar{e}^T(t)\left(\Lambda^2\otimes BB^T\right)\bar{e}(t)\\ & -\ 2\varpi(1-l)\tilde{x}^T(t)\left(L_1^2\otimes P^{-1}BB^TP^{-1}\right)\tilde{x}(t)\\ & +\ 2\sum_{i=1}^{M}(\alpha_i(t)-\varpi)(-\beta_i\alpha_i(t)).\end{aligned} \tag{21}$$

For the last term of (21), we make the following transformation:

$$\begin{aligned}& 2\sum_{i=1}^{M}(\alpha_i(t)-\varpi)(-\beta_i\alpha_i(t))\\ &< -\sum_{i=1}^{M}\beta_i\left(\alpha_i^2(t)-2\varpi\alpha_i(t)\right)\\ &= -\sum_{i=1}^{M}\beta_i\left(\alpha_i^2(t)-2\varpi\alpha_i(t)+\varpi^2-\varpi^2\right)\\ &= -\sum_{i=1}^{M}\beta_i(\alpha_i(t)-\varpi)^2+\sum_{i=1}^{M}\beta_i\varpi^2.\end{aligned} \tag{22}$$

Combining with (21) and (22), we have

$$\begin{aligned}\dot{V}(t) < {} & \sum_{i=1}^{M}\lambda_i\bar{e}_i^T(t)\left(AP+PA^T-2\varpi(1-\frac{1}{l})\lambda_iBB^T\right)\bar{e}_i(t)\\ & -\ 2\tilde{x}^T(t)\left(L_1\Psi(t)L_1\otimes P^{-1}BB^TP^{-1}\right)e(t)\\ & +\ 2\tilde{x}^T(t)\left(L_1\Psi(t)L_1\otimes P^{-1}BB^TP^{-1}\right)\tilde{x}(t)\\ & +\ 2\rho\tilde{x}^T(t)(I_M\otimes Q(A-GC))\tilde{x}(t)\\ & -\ 2\varpi(1-l)\tilde{x}^T(t)\left(L_1^2\otimes P^{-1}BB^TP^{-1}\right)\tilde{x}(t)\\ & -\ \sum_{i=1}^{M}\beta_i(\alpha_i(t)-\varpi)^2+\sum_{i=1}^{M}\beta_i\varpi^2.\end{aligned} \tag{23}$$

If ϖ is selected as $\varpi > (\eta/[2\lambda_{\min}(L_1)(1-[1/l])])$, then (23) is rewritten as follows:

$$\begin{aligned}\dot{V}(t) < {} & \sum_{i=1}^{M}\lambda_i\bar{e}_i^T(t)\left(AP+PA^T-\eta BB^T\right)\bar{e}_i(t)\\ & -\ 2\tilde{x}^T(t)\left(L_1\Psi(t)L_1\otimes P^{-1}BB^TP^{-1}\right)e(t)\\ & +\ 2\tilde{x}^T(t)\left(L_1\Psi(t)L_1\otimes P^{-1}BB^TP^{-1}\right)\tilde{x}(t)\\ & +\ 2\rho\tilde{x}^T(t)(I_M\otimes Q(A-GC))\tilde{x}(t)\\ & -\ 2\varpi(1-l)\tilde{x}^T(t)\left(L_1^2\otimes P^{-1}BB^TP^{-1}\right)\tilde{x}(t)\\ & -\ \sum_{i=1}^{M}\beta_i(\alpha_i(t)-\varpi)^2+\sum_{i=1}^{M}\beta_i\varpi^2.\end{aligned} \tag{24}$$

According to conditions (13) and (14) in Theorem 1, (24) is rewritten as follows:

$$\begin{aligned}\dot{V}(t) < {} & -\left(\theta_1+\tilde{\theta}_1\right)\sum_{i=1}^{M}\lambda_i\bar{e}_i^T(t)P\bar{e}_i(t)\\ & -\ 2\tilde{x}^T(t)\left(L_1\Psi(t)L_1\otimes P^{-1}BB^TP^{-1}\right)e(t)\\ & +\ 2\tilde{x}^T(t)\left(L_1\Psi(t)L_1\otimes P^{-1}BB^TP^{-1}\right)\tilde{x}(t)\end{aligned}$$

$$
\begin{aligned}
&- \left(\theta_2 + \tilde{\theta}_2\right)\rho\tilde{x}^T(t)(I_M \otimes Q)\tilde{x}(t) \\
&- 2\varpi(1-l)\tilde{x}^T(t)\left(L_1^2 \otimes P^{-1}BB^TP^{-1}\right)\tilde{x}(t) \\
&- \sum_{i=1}^M \beta_i(\alpha_i(t) - \varpi)^2 + \sum_{i=1}^M \beta_i\varpi^2 \\
=& -\theta_1 e^T(t)(L_1 \otimes P^{-1})e(t) - \rho\theta_2\tilde{x}^T(t)(I_M \otimes Q)\tilde{x}(t) \\
&- \sum_{i=1}^M \beta_i(\alpha_i(t) - \varpi)^2 + \sum_{i=1}^M \beta_i\varpi^2 + z_1^T(t)\Omega_1 z_1(t)
\end{aligned} \tag{25}
$$

where $z_1(t) = [e^T(t), \tilde{x}^T(t)]^T$ and

$$
\Omega_1 = \begin{bmatrix} -\tilde{\theta}_1(L_1 \otimes P^{-1}) & -(L_1\Psi(t)L_1 \otimes P^{-1}BB^TP^{-1}) \\ * & \Xi_1 \end{bmatrix}
$$

with $\Xi_1 = -\tilde{\theta}_2\rho(I_M \otimes Q) + 2(L_1\Psi(t)L_1 \otimes P^{-1}BB^TP^{-1}) - 2\varpi(1-l)(L_1^2 \otimes P^{-1}BB^TP^{-1})$.

Due to the fact that $-\tilde{\theta}_1(L_1 \otimes P^{-1}) < 0$, then $\Omega_1 < 0$ is equivalent to the following inequality by using the Schur complement lemma:

$$
\Xi_1 + \frac{1}{\tilde{\theta}_1}\left(L_1\Psi(t)L_1\Psi(t)L_1 \otimes P^{-1}BB^TP^{-1}BB^TP^{-1}\right) < 0.
$$

Moreover, we have

$$
\begin{aligned}
&2\left[(L_1\Psi(t)L_1 - \varpi(1-l)L_1^2) \otimes P^{-1}BB^TP^{-1}\right] \\
&\quad + \frac{1}{\tilde{\theta}_1}\left(L_1\Psi(t)L_1\Psi(t)L_1 \otimes P^{-1}BB^TP^{-1}BB^TP^{-1}\right) \\
&< 2\lambda_{\max}\left(L_1\bar{\Psi}L_1\right)\left(I_M \otimes P^{-1}BB^TP^{-1}\right) \\
&\quad - 2\varpi(1-l)\lambda_{\max}\left(L_1^2\right)\left(I_M \otimes P^{-1}BB^TP^{-1}\right) \\
&\quad + \frac{\lambda_{\max}\left(L_1\bar{\Psi}L_1\bar{\Psi}L_1\right)}{\tilde{\theta}_1}\left(I_M \otimes P^{-1}BB^TP^{-1}BB^TP^{-1}\right)
\end{aligned}
$$

where $\bar{\Psi} = \mathrm{diag}\{\bar{\alpha}_1, \ldots, \bar{\alpha}_M\}$.

Then, we can select a sufficiently large positive constant ρ such that

$$
\begin{aligned}
&-\tilde{\theta}_2\rho Q + 2\lambda_{\max}\left(L_1\bar{\Psi}L_1\right)P^{-1}BB^TP^{-1} \\
&\quad - 2\varpi(1-l)\lambda_{\max}\left(L_1^2\right)P^{-1}BB^TP^{-1} \\
&\quad + \frac{\lambda_{\max}\left(L_1\bar{\Psi}L_1\bar{\Psi}L_1\right)}{\tilde{\theta}_1}P^{-1}BB^TP^{-1}BB^TP^{-1} < 0
\end{aligned}
$$

which implies $\Omega_1 < 0$. Then, we can get

$$
\begin{aligned}
\dot{V}(t) &< -\theta_1 e^T(t)\left(L_1 \otimes P^{-1}\right)e(t) - \rho\theta_2\tilde{x}^T(t)(I_M \otimes Q)\tilde{x}(t) \\
&\quad - \sum_{i=1}^M \beta_i(\alpha_i(t) - \varpi)^2 + \sum_{i=1}^M \beta_i\varpi^2 \\
&< -\xi_1\Bigg[e^T(t)\left(L_1 \otimes P^{-1}\right)e(t) + \rho\tilde{x}^T(t)(I_M \otimes Q)\tilde{x}(t) \\
&\quad + \sum_{i=1}^M \frac{1}{\kappa_i}(\alpha_i(t) - \varpi)^2\Bigg] + \sum_{i=1}^M \beta_i\varpi^2 \\
&< -\xi_1 V(t) + \Upsilon
\end{aligned} \tag{26}
$$

where $\xi_1 = \min\{\theta_1, \theta_2, \beta_i\kappa_i\}$ and $\Upsilon = \sum_{i=1}^M \beta_i\varpi^2$.

Multiplying $e^{\xi_1 t}$ and taking the integral from the both sides of (26) leads to $\int_{T_{n-1}^{\text{off}}}^t \dot{V}(t)e^{\xi_1 t}dt < -\int_{T_{n-1}^{\text{off}}}^t \xi_1 e^{\xi_1 t}V(t)dt + \int_{T_{n-1}^{\text{off}}}^t \Upsilon e^{\xi_1 t}dt$. Then, we have

$$
V(t) < V\left(T_{n-1}^{\text{off}}\right)e^{-\xi_1\left(t - T_{n-1}^{\text{off}}\right)} + \int_{T_{n-1}^{\text{off}}}^t \Upsilon dt \tag{27}
$$

where the following transformations are used:

$$
\begin{aligned}
\int_{T_{n-1}^{\text{off}}}^t e^{\xi_1 t}d(V(t)) &< -\int_{T_{n-1}^{\text{off}}}^t \xi_1 e^{\xi_1 t}V(t)dt \\
&\quad + \int_{T_{n-1}^{\text{off}}}^t \Upsilon e^{\xi_1 t}dt \\
V(t)e^{\xi_1 t}\Big|_{T_{n-1}^{\text{off}}}^t - \int_{T_{n-1}^{\text{off}}}^t \xi_1 e^{\xi_1 t}V(t)dt &< -\int_{T_{n-1}^{\text{off}}}^t \xi_1 e^{\xi_1 t}V(t)dt \\
&\quad + \int_{T_{n-1}^{\text{off}}}^t \Upsilon e^{\xi_1 t}dt \\
V(t)e^{\xi_1 t} - V\left(T_{n-1}^{\text{off}}\right)e^{\xi_1 T_{n-1}^{\text{off}}} &< \int_{T_{n-1}^{\text{off}}}^t \Upsilon e^{\xi_1 t}dt.
\end{aligned}
$$

In the presence of DoS attacks, i.e., $t \in [T_n^{\text{on}}, T_n^{\text{off}})$, we can get

$$
\begin{aligned}
\dot{V}(t) = & 2e^T(t)\left(L_1 \otimes P^{-1}A\right)e(t) \\
&- 2e^T(t)\left(L_1\Psi(t) \otimes P^{-1}BB^TP^{-1}\right)\tilde{\delta}(t) \\
&+ 2\rho\tilde{x}^T(t)(I_M \otimes Q(A - GC))\tilde{x}(t) \\
&+ 2\sum_{i=1}^M \alpha_i(t)\tilde{\delta}_i^T(t)P^{-1}BB^TP^{-1}\tilde{\delta}_i(t) \\
&- 2\varpi\sum_{i=1}^M \tilde{\delta}_i^T(t)P^{-1}BB^TP^{-1}\tilde{\delta}_i(t) \\
&+ 2\sum_{i=1}^M(\alpha_i(t) - \varpi)(-\beta_i\alpha_i(t)).
\end{aligned} \tag{28}
$$

According to conditions (14) and (15) in Theorem 1, we can get

$$
\begin{aligned}
\dot{V}(t) < & \left(\theta_3 - \tilde{\theta}_3\right)e^T(t)\left(L_1 \otimes P^{-1}\right)e(t) \\
&- 2e^T(t)\left(L_1\Psi(t) \otimes P^{-1}BB^TP^{-1}\right)\tilde{\delta}(t) \\
&- \left(\theta_2 + \tilde{\theta}_2\right)\rho\tilde{x}^T(t)(I_M \otimes Q)\tilde{x}(t) \\
&+ 2\sum_{i=1}^M \alpha_i(t)\tilde{\delta}_i^T(t)P^{-1}BB^TP^{-1}\tilde{\delta}_i(t) \\
&- 2\varpi\sum_{i=1}^M \tilde{\delta}_i^T(t)P^{-1}BB^TP^{-1}\tilde{\delta}_i(t) \\
&+ 2\sum_{i=1}^M(\alpha_i(t) - \varpi)(-\beta_i\alpha_i(t)).
\end{aligned} \tag{29}
$$

From (22), we have $2\sum_{i=1}^M(\alpha_i(t) - \varpi)(-\beta_i\alpha_i(t)) < -\sum_{i=1}^M \beta_i(\alpha_i(t) - \varpi)^2 + \sum_{i=1}^M \beta_i\varpi^2$.

Then, we can get

$$\begin{aligned}\dot{V}(t) &< \theta_3 e^T(t)(L_1 \otimes P^{-1})e(t)\\ &\quad - \left(\theta_2 + \tilde{\theta}_2\right)\rho \tilde{x}^T(t)(I_M \otimes Q)\tilde{x}(t)\\ &\quad - \sum_{i=1}^{M} \beta_i(\alpha_i(t) - \varpi)^2 + \sum_{i=1}^{M} \beta_i \varpi^2 + z_2^T(t)\Omega_2 z_2(t)\end{aligned}$$

where $z_2(t) = [e^T(t), \tilde{\delta}^T(t)]^T$ and

$$\Omega_2 = \begin{bmatrix} -\tilde{\theta}_3(L_1 \otimes P^{-1}) & -(L_1\Psi(t) \otimes P^{-1}BB^T P^{-1}) \\ * & \Xi_2 \end{bmatrix}$$

with $\Xi_2 = \sum_{i=1}^{M} 2(\alpha_i(t) - \varpi)P^{-1}BB^T P^{-1}$.

Similarly, since $-\tilde{\theta}_3(L_1 \otimes P^{-1}) < 0$, then $\Omega_2 < 0$ is equivalent to

$$\Xi_2 + \frac{1}{\tilde{\theta}_3}\left(L_1\Psi^2(t) \otimes P^{-1}BB^T P^{-1}BB^T P^{-1}\right) < 0.$$

Moreover, we can get $(1/\tilde{\theta}_3)(L_1\Psi^2(t) \otimes P^{-1}BB^T P^{-1}BB^T P^{-1}) < (1/\tilde{\theta}_3)\lambda_{\max}(L_1\bar{\Psi}^2)(I_M \otimes P^{-1}BB^T P^{-1}BB^T P^{-1})$. Then, we can choose a sufficiently large ϖ (large than $(\eta/[2\lambda_{\min}(L_1)(1 - [1/l])])$) such that

$$\begin{aligned}&-2\varpi P^{-1}BB^T P^{-1} + 2\bar{\alpha}_i P^{-1}BB^T P^{-1}\\ &\quad + \frac{1}{\tilde{\theta}_3}\lambda_{\max}(L_1\bar{\Psi}^2)P^{-1}BB^T P^{-1}BB^T P^{-1} < 0\end{aligned}$$

which implies $\Omega_2 < 0$. Then, we have

$$\begin{aligned}\dot{V}(t) &< \theta_3 e^T(t)\left(L_1 \otimes P^{-1}\right)e(t)\\ &\quad - \left(\theta_2 + \tilde{\theta}_2\right)\rho \tilde{x}^T(t)(I_M \otimes Q)\tilde{x}(t)\\ &\quad - \sum_{i=1}^{M} \beta_i(\alpha_i(t) - \varpi)^2 + \sum_{i=1}^{M} \beta_i \varpi^2\\ &< \xi_2 V(t) + \Upsilon \end{aligned} \tag{30}$$

where $\xi_2 = \max\{\theta_3, -(\theta_2 + \tilde{\theta}_2), -\beta_i\kappa_i\}$.

Similar to (27), we have

$$V(t) < V\left(T_n^{\text{on}}\right)e^{\xi_2(t - T_n^{\text{on}})} + \int_{T_n^{\text{on}}}^{t} \Upsilon dt. \tag{31}$$

Considering two cases of DoS attacks and combining with (27) and (31), we can get

$$V(t) < \begin{cases} V\left(T_{n-1}^{\text{off}}\right)e^{-\xi_1\left(t - T_{n-1}^{\text{off}}\right)} + \int_{T_{n-1}^{\text{off}}}^{t} \Upsilon dt, & t \in \left[T_{n-1}^{\text{off}}, T_n^{\text{on}}\right) \\ V\left(T_n^{\text{on}}\right)e^{\xi_2(t - T_n^{\text{on}})} + \int_{T_n^{\text{on}}}^{t} \Upsilon dt, & t \in \left[T_n^{\text{on}}, T_n^{\text{off}}\right). \end{cases}$$

For the case of $t \in [T_{n-1}^{\text{off}}, T_n^{\text{on}})$, we have

$$\begin{aligned}V(t) &< V\left(T_{n-1}^{\text{off}}\right)e^{-\xi_1\left(t - T_{n-1}^{\text{off}}\right)} + \int_{T_{n-1}^{\text{off}}}^{t} \Upsilon dt\\ &< V\left(T_{n-1}^{\text{on}}\right)e^{\xi_2\left(T_{n-1}^{\text{off}} - T_{n-1}^{\text{on}}\right)}e^{-\xi_1\left(t - T_{n-1}^{\text{off}}\right)}\\ &\quad + \int_{T_{n-1}^{\text{off}}}^{t} \Upsilon dt + \int_{T_{n-1}^{\text{on}}}^{T_{n-1}^{\text{off}}} \Upsilon e^{-\xi_1\left(t - T_{n-1}^{\text{off}}\right)} dt\\ &< \cdots\\ &< V(0)e^{-\xi_1|\Xi_s(0,t)|}e^{\xi_2|\Xi_a(0,t)|}\\ &\quad + \int_0^t \Upsilon e^{-\xi_1|\Xi_s(0,t)|}e^{\xi_2|\Xi_a(0,t)|} dt. \end{aligned} \tag{32}$$

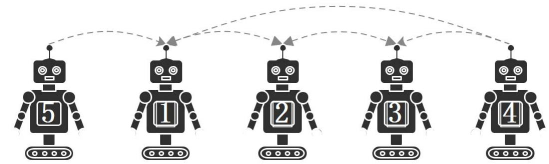

Fig. 2. Topology graph $\mathcal{G}$ with five agents.

Similar to the case of $t \in [T_{n-1}^{\text{off}}, T_n^{\text{on}})$, we have the following inequality for the case of $t \in [T_n^{\text{on}}, T_n^{\text{off}})$:

$$\begin{aligned}V(t) &< V\left(T_n^{\text{on}}\right)e^{\xi_2(t - T_n^{\text{on}})} + \int_{T_n^{\text{on}}}^{t} \Upsilon dt\\ &< V\left(T_{n-1}^{\text{off}}\right)e^{-\xi_1\left(T_n^{\text{on}} - T_{n-1}^{\text{off}}\right)}e^{\xi_2(t - T_n^{\text{on}})}\\ &\quad + \int_{T_n^{\text{on}}}^{t} \Upsilon dt + \int_{T_{n-1}^{\text{off}}}^{T_n^{\text{on}}} \Upsilon e^{\xi_2(t - T_n^{\text{on}})} dt\\ &\leq \cdots\\ &< V(0)e^{-\xi_1|\Xi_s(0,t)|}e^{\xi_2|\Xi_a(0,t)|}\\ &\quad + \int_0^t \Upsilon e^{-\xi_1|\Xi_s(0,t)|}e^{\xi_2|\Xi_a(0,t)|} dt. \end{aligned} \tag{33}$$

From (9), (10), and (12), we have $|\Xi_s(0,t)| = t - 0 - |\Xi_a(0,t)|$ and $|\Xi_a(0,t)| < \Xi_0 + (t/\tau_a)$. Combining with (32) and (33), we have

$$\begin{aligned}V(t) &< e^{-\xi_1(t - |\Xi_a(0,t)|)}e^{\xi_2|\Xi_a(0,t)|}V(0)\\ &\quad + \int_0^t \Upsilon e^{-\xi_1(t - |\Xi_a(0,t)|)}e^{\xi_2|\Xi_a(0,t)|} dt\\ &< e^{(\xi_1+\xi_2)\Xi_0}e^{-\left(\xi_1 - \frac{\xi_1+\xi_2}{\tau_a}\right)t}V(0)\\ &\quad + \int_0^t \Upsilon e^{(\xi_1+\xi_2)\Xi_0}e^{-\left(\xi_1 - \frac{\xi_1+\xi_2}{\tau_a}\right)t} dt. \end{aligned} \tag{34}$$

According to condition (16) in Theorem 1, we can get $\tau_a > (\xi_1 + \xi_2/\xi_1)$, which together with (34) implies that $V(t)$ is uniformly bounded when $t \to \infty$, that is, $V(t)$ converges the following set:

$$\left\{V(t) \mid \|V(t)\| < e^{(\xi_1+\xi_2)\Xi_0}(V(0) + \Upsilon)\right\}.$$

■

Remark 6: Notice that the use of the information of the whole network topology structure, i.e., $\lambda_{\min}(L_1)$, can be avoided in the control scheme by introducing the constant ϖ and the adaptive parameter $\alpha_i(t)$. Although ϖ is related to $\lambda_{\min}(L_1)$, ϖ is not included in the control scheme and is only introduced in the process of the stability analysis.

IV. Simulation

This section aims to use the mobile robot system [41] to testify the validity of the proposed control scheme. Assume that there exist three followers labeled as 1–3 and 2 leaders marked by 4 and 5, and the topology graph is shown in Fig. 2. The linearized dynamic of the ith agent is as follows [41]:

$$\begin{aligned}\dot{x}_i(t) &= Ax_i(t) + Bu_i(t)\\ y(t) &= Cx_i(t)\end{aligned}$$

where $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

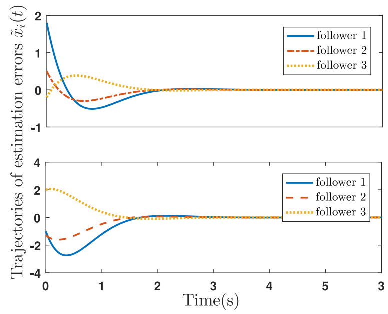

Fig. 3. Trajectories of estimation errors $\tilde{x}_i(t)$.

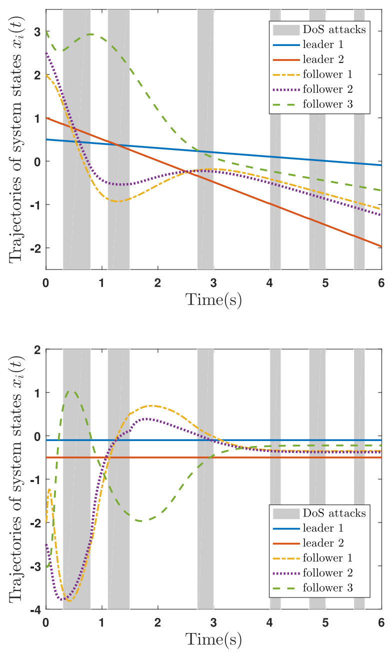

Fig. 4. Trajectories of system states $x_i(t)$.

Select $\theta_1 = 2$, $\tilde{\theta}_1 = 0.1$, $\theta_2 = 2$, $\tilde{\theta}_2 = 0.1$, $\eta = 2.6$, $\theta_3 = 4.9$, $\tilde{\theta}_3 = 0.4$, $\kappa_i = 2$, $\beta_i = 1$, $\bar{\alpha}_i = 10$, and $\underline{\alpha}_i = 0$. The observer and controller gains are obtained as $G = [3.5099, 6.0864]^T$ and $K = [2.7515, 2.2468]$, respectively. According to inequality (16) in Theorem 1, τ_a needs to be satisfy $\tau_a > (2 + 4.9/2) = 3.45$. We choose $\tau_a = 3.5$ and $\Xi_0 = 0.2$, then $|\Xi_a(0, 6)| = 0.2 + 1.714 = 1.914$. Select the initial values as $\hat{x}_f(0) = [0, -1, 1.5, -1, 5, -5]^T$, $x_f(0) = [2, -2, 2.5, -2.5, 3, -3]^T$, and $x_L(0) = [0.5, -0.1, 1, -0.5]^T$.

We can observe from Fig. 3 that the devised observer can estimate system states well. Furthermore, we can observe from Fig. 4 that the containment control objective can be accomplished by using the designed attack-resilient control scheme, which is also observed from the trajectories of containment errors $e_i(t)$ shown in Fig. 5. The trajectories of adaptive parameters $\alpha_i(t)$ are presented in Fig. 6.

We further make a comparison between the existing control scheme designed in [37] and the proposed one to further highlight the validity of the proposed control scheme against DoS attacks. The comparison results are shown in Figs. 7 and 8. From Fig. 7, we can observe that the containment control task can be accomplished by using the control scheme designed in [37] or ours. But the containment error obtained by using the proposed control scheme is smaller than that in [37], which can also be observed in Table I. This implies that the effect of resisting attacks by using the proposed control scheme is better. Additionally, although the attack compensation mechanism designed in [37] can resist DoS attacks to some extent, it will be invalid when the duration of DoS attacks is greater than a certain value, which is illustrated in Fig. 8. We can observe from Fig. 8 that system states with the attack compensation mechanism designed in [37] is unbounded, while the containment control task can still be accomplished by using the devised attack-resilient control scheme in our paper.

Moreover, an unstable numerical example is further presented to verify the effectiveness of the devised control scheme. The system equation of the ith agent is as following:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t)$$
$$y(t) = Cx_i(t)$$

where $A = \begin{bmatrix} 0.5 & 1 \\ -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$. Besides, the rest of parameters are the same as the previous simulation. The simulation results are illustrated in Figs. 9 and 10, and

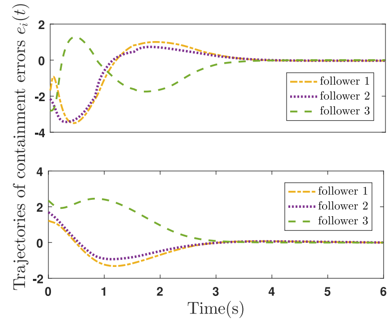

Fig. 5. Trajectories of containment errors $e_i(t)$.

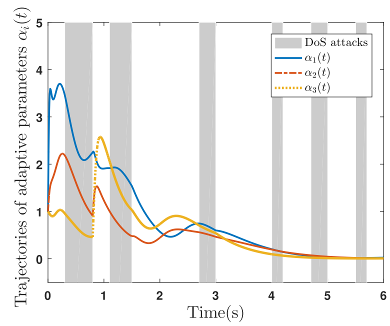

Fig. 6. Trajectories of adaptive parameters $\alpha_i(t)$.

TABLE I
2-Norm of Containment Errors

Containment error	Ours	[37]
$\|e_1(t)\|_2$	28.1271	35.9203
$\|e_2(t)\|_2$	28.2283	30.8426
$\|e_3(t)\|_2$	21.6695	30.8389

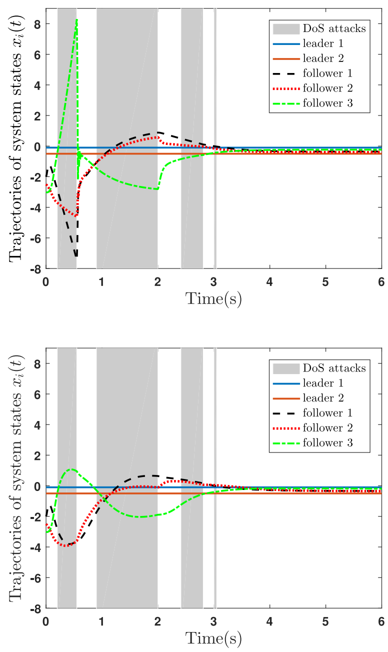

Fig. 7. Comparison between [37] methods (up) and ours methods (down).

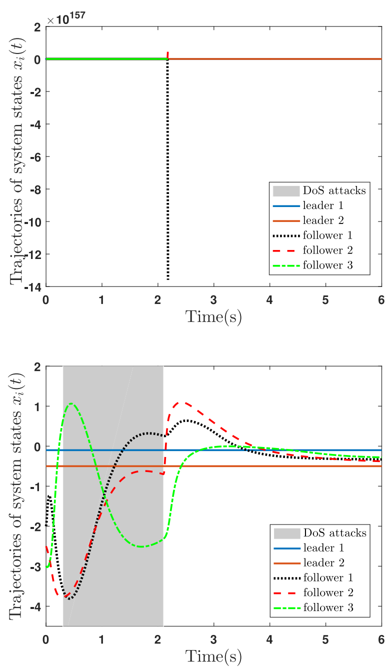

Fig. 8. Comparison between [37] methods (up) and ours methods (down).

we can observe that the containment control task can still be completed in the case of the unstable system.

V. Conclusion

The fully distributed containment control problem subject to DoS attacks is addressed in this article. The fully distributed containment control framework is established under DoS attacks. A fully distributed attack-resilient controller with an observer is devised to accomplish the states containment task, in which the knowledge of the whole network topology is not required. Besides, a novel attack compensator is devised

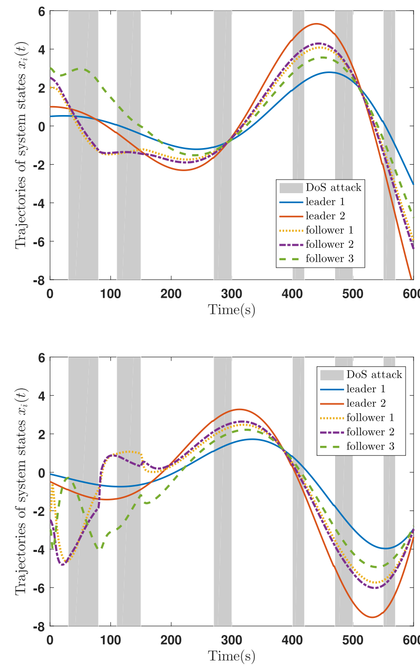

Fig. 9. Trajectories of system states $x_i(t)$.

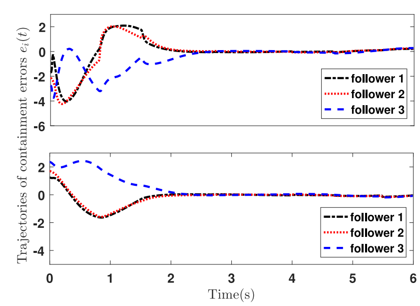

Fig. 10. Trajectories of containment errors $e_i(t)$.

to mitigate the impact of DoS attacks. Finally, the simulation results present that the devised controller with an attack compensation mechanism can effectively resist to DoS attacks by a comparison.

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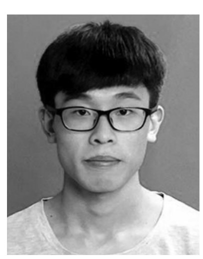

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