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Observer-Based Event-Triggered Containment Control for MASs Under DoS Attacks

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***Abstract*—This article studies the observer-based event-triggered containment control problem for linear multiagent systems (MASs) under denial-of-service (DoS) attacks. In order to deal with situations where MASs states are unmeasurable, an improved separation method-based observer design method with less conservativeness is proposed to estimate MASs states. To save communication resources and achieve the containment control objective, a novel observer-based event-triggered containment controller design method based on observer states is proposed for MASs under the influence of DoS attacks, which can make the MASs resilient to DoS attacks. In addition, the Zeno behavior can be eliminated effectively by introducing a positive constant into the designed event-triggered mechanism. Finally, a practical example is presented to illustrate the effectiveness of the designed observer and the event-triggered containment controller.**

***Index Terms*—Containment control, denial-of-service (DoS) attacks, event-triggered mechanism, multiagent systems (MASs).**

I. Introduction

RECENTLY, the cooperative control problem of multiagent systems (MASs) has aroused great research interest due to the wide application, such as remote unmanned aerial vehicles (UAVs) [1], multirobot systems [2], etc. Compared with the traditional control system, MASs have the advantages of distribution, cooperation, autonomy, fault tolerance, high efficiency, and low cost. Containment control, as a method of the multiagent cooperative control, has attracted remarkable attention and research [3]–[6]. In containment control, all agents are divided into followers and leaders. The main target of the containment control is to design a controller to enable a group of followers to reach and remain on a convex hull formed and guided by multiple leaders. In the practical application, containment control plays a key role in the coordination tasks, such as hazardous material handling, fire rescue, and cooperative transportation. The containment control problem for linear MASs under aperiodic sampling interval and measurement size reduction is investigated in [4]. In [5], the problem of the fault-containment control for a kind of heterogeneous linear MASs is studied, and the effect of system failures is mitigated by developing a fault-tolerant hierarchical containment control protocol. The output containment control problem is investigated for heterogeneous linear MASs in [6], and an optimal control law that can achieve the output containment control is obtained by an algebraic Riccati equation.

Notice that the network communication considered in the above-mentioned literature is continuous, which inevitably leads to the waste of communication resources. To save communication resources, an event-triggered mechanism is introduced to reduce the number of network communications [7]–[17]. Ding *et al.* [7] provided an overview of the event-triggered control for MASs, in which the existing four types of event-triggered schemes are introduced, respectively. Wang *et al.* [8] studied the event-triggered containment control problem for linear MASs. The event-triggered H_∞ containment control problem for discrete time-varying linear MASs is investigated in [9]. The problem of the distributed adaptive fuzzy event-triggered containment control for a class of nonlinear strict-feedback systems is studied in [10]. In [12], the containment control problems based on the event-triggered output feedback for homogeneous and heterogeneous MASs are investigated, respectively. The problem of the consensus using the dynamic event-triggered approach for heterogeneous MASs with time delays is investigated in [15]. However, using the event-triggered mechanism may result in the Zeno behavior. The Zeno behavior means that an infinite number of events happen in a finite-time interval, which results in the frequent communication between the system and the controller center, so the computational and communication resources are greatly wasted. Therefore, it is important to eliminate the Zeno behavior both for theoretical and practical significance. A decaying function is added into the event-triggered function to exclude the Zeno behavior [18], but this will make the stability analysis of the closed-loop system more complex. Therefore, a novel hybrid event-triggered mechanism is first proposed in [19] to

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solve the problem of the Zeno behavior. The main advantage of the hybrid event-triggered mechanism is that the interevent time is at least bounded from below by a strictly positive constant by introducing a positive constant into the event-triggered condition and it will not bring the extra complexity to the stability analysis of the closed-loop system.

It is worth noting that agents usually communicate with each other through a wireless network. It is noted that the aforementioned research assumes that the communication channels used to transmit data packets are perfect. However, some phenomena are inevitable in the actual system, such as malicious network attacks, packet losses, disorder, network-induced delays, sensor or actuator faults, and so on. Therefore, many research works have been contributed to this problem. It should be pointed out that the observer can be used to solve the problems of unmeasurable system states and sensor or actuator faults. Lee *et al.* [20] addressed the observer-based fault-tolerant H_∞ control problem, and design observers to estimate both system states and faults. In [21], a sliding-mode observer is designed to reconstruct faults. In [22], the robust adaptive fault-tolerant control circuit design problem is investigated. In addition, the problem of the resilient containment control for MASs with packet dropouts is investigated in [23]. In [24], the problem of the event-triggered containment control based on observer states for MASs with time delay is investigated. Wang and Wang [25] provided the necessary and sufficient conditions that ensure the containment control can be achieved for MASs with time delay. Besides, the problem of the containment control for type-2 fuzzy networked MASs with packet dropouts and actuator faults is investigated in [26].

But for all the above-mentioned research, a serious problem of cyber attacks has not been explored. It is well known to all that malicious network attacks are inevitable and have a serious impact on MASs. Thus, it is urgent and necessary to study MASs with the consideration of malicious network attacks. There are usually two kinds of malicious network attacks: 1) denial-of-service (DoS) [27], [28] and 2) deception attacks [29]–[31]. Deception attacks aim to destroy the system performance by changing the data integrity, while DoS attacks are to block data transmission. When there are DoS attacks in the system, the system will be divided as a class of switched systems consisted of a closed-loop subsystem and an open-loop subsystem. Several pioneering works about switched systems have been studied in [32]–[34]. In addition, there have been some recent results of the consensus control for MASs with the consideration of cyber attacks [35]–[39]. In [35], the event-triggered consensus control problem for a class of nonlinear MASs in the case of DoS attacks is investigated. Ding *et al.* [36] investigated the problem of the observer-based event-triggered consensus control for discrete-time MASs subjected to false data-injection attacks and lossy sensors. In addition, the problem of the event-triggered consensus control for nonlinear second-order MASs in the case of DoS attacks is investigated in [38]. Feng and Hu [39] investigated the event-triggered consensus control problem for linear MASs in the case of DoS attacks. It is noted that the open-loop observer is used to design the event-triggered distributed controller in [37]–[39]. However, the open-loop observer will work only when the initial state of the observer equals the initial state of the system. But, it is very difficult to guarantee this point in the practical system. Thus, it is necessary to design an observer-based event-triggered distributed controller in consideration of unmeasurable system states.

Most of the existing literature focuses on the consensus control problem under cyber attacks [35]–[39] and there are few results related to the observer-based event-triggered containment control problem under cyber attacks. In [40] and [41], the resilient containment control problems for MASs subjected to FDI attacks are investigated, respectively. The problem of the fuzzy containment control under DoS attacks is studied in [42], in which the bilinear term that existed in controller design conditions is eliminated by using the cone complementarity linearization (CCL) algorithm. However, as mentioned in [43], the CCL algorithm is not always possible to find the global optimal solution. In addition, the singular value decomposition (SVD) and orthogonal basis methods are used in the observer-based event-triggered controller design in [36] and [44], but the control input matrix is assumed to be full-column rank. Therefore, it is necessary to develop a novel resilient observer-based event-triggered containment controller design method with the less conservatism.

Motivated by the formerly mentioned problems and in order to solve them, namely, to save network resources and guarantee the reliability of MASs at the same time, this article studies the observer-based event-triggered containment control problem for MASs under DoS attacks, which still lacks relevant research to the authors' knowledge. First, the distributed containment control protocol composed of an observer and an observer-based event-triggered state-feedback controller is modeled for MASs with the consideration of DoS attacks. Then, an observer-based distributed event-triggered containment controller is designed to achieve the states containment by using the linear matrix inequality (LMI) technique. A practical UAV example is presented to illustrate the effectiveness of the proposed method by comparing with the existing method. The main contributions of this article are summarized as follows.

1) First, a new observer-based event-triggered containment control framework with the consideration of DoS attacks is established. Then, a novel resilient event-triggered containment controller design method based on the designed observer is proposed to achieve the states containment for MASs under DoS attacks, in which the frequency and duration of DoS attacks obtained in this article are only related to a few positive constants. Besides, the Zeno behavior can be eliminated effectively by introducing a positive constant into the event-triggered mechanism.
2) In the existing observer-based event-triggered controller design method [36], [44], the SVD and orthogonal basis methods are applied, which assumes that the control input matrix is full-column rank. Different from them, an improved separation method is proposed in the observer-based event-triggered controller design in this article, and the assumption of the full-column rank is removed. Then, the nonconvex design conditions can be converted

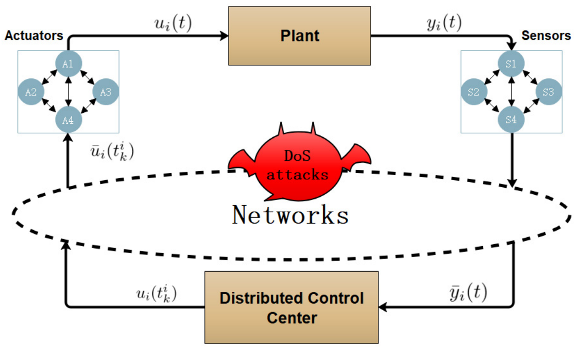

Fig. 1. System block diagram of the MAS under DoS attacks.

into the convex ones. Further, the controller and observer gains can be solved by using the LMI technique.

The outline of this article is as follows. First, in Section II, some preliminaries and problem formulation are provided. Second, in Section III, the stability analysis of the containment control is presented. The design method of the observer-based event-triggered containment controller is given in Section IV. In Section V, an example is given to illustrate the effectiveness of the proposed method by comparing it with the traditional method. Finally, conclusions are presented in Section VI.

Notations: The asterisk $*$ represents the symmetry term in the matrix. I_M represents the M-dimensional identity matrix and 0_M represents the M-dimensional zero matrix. $\|\cdot\|$ represents the Euclidean norm. $H > 0$ indicates H is a positive-definite matrix. $\otimes$ stands the Kronecker product. $\mathbb{R}^n$ denotes the n-dimensional real number space. $\mathbb{R}^{n\times n}$ denotes the set of $n \times n$-dimensional real matrices. $M \setminus N$ denotes the set belongs to the set M but not the set N. diag$\{\cdot\}$ stands for the diagonal matrix. $\mathbb{N}$ represents the positive real number.

II. Problem Formulation And Preliminaries

This section aims to model linear MASs with the distributed containment control protocol composed of an observer and an observer-based event-triggered state-feedback controller in the case of DoS attacks. The system block diagram of the MAS under DoS attacks is illustrated in Fig. 1.

In Fig. 1, $y_i(t)$, $u_i(t)$, and $u_i(t_k^i)$ are the measurement output, the control input, and the control input at time instant t_k^i, respectively. $\bar{y}_i(t)$ and $\bar{u}_i(t_k^i)$ represent the measurement output and the control input under DoS attacks, respectively.

A. Preliminaries

Some knowledge of graph theory is demonstrated. The topology graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ represents the interaction among MASs, where $\mathcal{V} = \{1, \ldots, N\}$ is the set of nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denotes the set of edges. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is represented as $a_{ii} = 0$ and $a_{ij} = 1$ if $(i, j) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. The neighborhood of the agent i is defined as $N_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}\}$ and the self-edge (i, i) satisfies $(i, i) \notin \mathcal{E}$. The in-degree matrix is defined as $\mathcal{D} = \mathrm{diag}\{d_1, \ldots, d_N\} \in \mathbb{R}^{N\times N}$ with $d_i = \sum_{j\in N_i} a_{ij}$. Then, the Laplacian matrix of the topology graph $\mathcal{G}$ is defined as $L = \mathcal{D} - \mathcal{A}$.

In this article, it is supposed that the number of followers among all agents is M and the followers are labeled by $1, \ldots, M$, while others are the leaders represented by $M + 1, \ldots, N$. Define $\mathcal{F} \triangleq \{1, \ldots, M\}$ and $\Omega \triangleq \{M + 1, \ldots, N\}$. A node is said to be a leader if the node has no communication with others and it is a follower otherwise. Therefore, the following Laplacian matrix can be obtained:

$$L = \begin{bmatrix} L_1 & L_2 \\ 0_{(N-M)\times M} & 0_{(N-M)\times(N-M)} \end{bmatrix}$$

where $L_1 \in \mathbb{R}^{M\times M}$ represents the interaction among all followers, while $L_2 \in \mathbb{R}^{M\times(N-M)}$ denotes the connection between followers and leaders.

The following assumption, lemma, and definition will be used in this article.

Assumption 1: It is assumed that there is at least one path connected to the leader for each follower.

Remark 1: As mentioned in the existing containment control results [23]–[26], Assumption 1 is introduced to guarantee that each follower can receive information from at least one leader. This assumption imposes the necessary condition to ensure that the containment control objective can be achieved. In fact, if there exist some followers isolated from other leaders, then the states containment for the isolated followers will never be achieved because they lose their reference signals.

Lemma 1 [46]: In the case of Assumption 1, L_1 is positive definite. Each element of $-L_1^{-1}L_2$ is non-negative and the sum of each row equals one. Therefore, one can use the orthogonal transform with the orthogonal matrix S such that

$$S^T L_1 S = \mathrm{diag}\{\lambda_1, \ldots, \lambda_M\} \triangleq \Lambda$$

where $\lambda_i (i = 1, \ldots, M)$ refer to the eigenvalues of L_1 and it is supposed that $\lambda_1 \geq \ldots \geq \lambda_M > 0$.

Definition 1 [47]: The containment control is said to be achieved for MASs, if there are some non-negative scalars $\chi_{ij} (i = 1, \ldots, M, j = M + 1, \ldots, N)$ with $\sum_{j=M+1}^{N} \chi_{ij} = 1$ such that

$$\lim_{t\to\infty} \left[x_i(t) - \sum_{j=M+1}^{N} \chi_{ij} x_j(t) \right] = 0.$$

B. MASs Modeling and Controller Structure Design

Consider the following linear MAS for the ith follower:

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t) \\ y_i(t) = Cx_i(t), \quad i \in \mathcal{F} \end{cases} \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$, $u_i(t) \in \mathbb{R}^m$, and $y_i(t) \in \mathbb{R}^d$ are the system state, the control input, and the measurement output for the ith follower, respectively. A, B, and C are given constant matrices with rational dimensions. It is assumed that the pairs (A, B) and (A, C) are controllable and observable, respectively.

The dynamics of the jth leader is defined as

$$\dot{x}_j(t) = Ax_j(t), j \in \Omega \tag{2}$$

where $x_j(t)$ is the state of the jth leader.

Remark 2: The role of leaders is to provide reference signals to followers. Similar to [9], [23], and [24], the control input of the leader system (2) is not considered in this article.

In the following part, a containment control protocol composed of an observer and an observer-based event-triggered state-feedback controller is designed. It is noted that not all states of agent are available in practice. In order to overcome this problem, the observer is designed as

$$\begin{cases} \dot{\hat{x}}_i(t) = A\hat{x}_i(t) + Bu_i(t) + G\big(y_i(t) - \hat{y}_i(t)\big) \\ \hat{y}_i(t) = C\hat{x}_i(t), \quad i \in \mathcal{F} \end{cases} \tag{3}$$

where $\hat{x}_i(t) \in \mathbb{R}^n$ and $\hat{y}_i(t) \in \mathbb{R}^d$ are the observer state and the estimated output, respectively. $G \in \mathbb{R}^{n\times d}$ is the observer gain to be designed later.

To achieve the containment control objective, the distributed controller is designed as

$$\begin{aligned} u_i(t) = K &\sum_{j\in N_i\cap\mathcal{F}} a_{ij}\big(\hat{x}_i(t) - \hat{x}_j(t)\big) \\ &+ K \sum_{j\in N_i\cap\Omega} a_{ij}\big(\hat{x}_i(t) - x_j(t)\big), \quad i \in \mathcal{F} \end{aligned} \tag{4}$$

where $K \in \mathbb{R}^{m\times n}$ is the controller gain to be designed later.

Obviously, the above designed control signal is continuous communication, which will waste network resources. To solve this problem and achieve the states containment at the same time, the event-triggered method is introduced to update the control signal. Suppose that the set of event-triggered instants for the ith agent is determined as $\{t_k^i, k = 0, 1, \ldots\}$. The event-triggered function is developed as follows:

$$f_i(t) = \|m_i(t)\| - \theta_i \big\|y_i(t) - \hat{y}_i(t)\big\|, \quad i \in \mathcal{F} \tag{5}$$

where θ_i is a positive constant to be selected rationally. $m_i(t) = q_i(t) - q_i(t_k^i)$ is the measurement error with $q_i(t_k^i)$ being the value at the latest event-triggered time t_k^i. $q_i(t) = \sum_{j\in N_i\cap\mathcal{F}} a_{ij}(\hat{x}_i(t) - \hat{x}_j(t)) + \sum_{j\in N_i\cap\Omega} a_{ij}(\hat{x}_i(t) - x_j(t))$ and $q_i(t_k^i) = \sum_{j\in N_i\cap\mathcal{F}} a_{ij}(\hat{x}_i(t_k^i) - \hat{x}_j(t_{k'}^j)) + \sum_{j\in N_i\cap\Omega} a_{ij}(\hat{x}_i(t_k^i) - x_j(t))$ with $\hat{x}_j(t_{k'}^j)$ being the jth agent observer state at the latest event-triggered instant $t_{k'}^j$. The control signal will be updated at the event-triggered instant t_k^i, which will be generated by the following hybrid event-triggered mechanism:

$$t_{k+1}^i = t_k^i + \Delta_k^i \tag{6}$$

where $\Delta_k^i = \max\{\tau_k^i, b_i\}$ is the interevent time. b_i is a positive constant to be determined and $\tau_k^i = \inf_{t>t_k^i}\{t - t_k^i | f_i(t) > 0\}$.

Remark 3: It can be observed that the above event-triggered function consists of two parts: 1) the measurement error $m_i(t)$ and 2) the containment error $q_i(t)$. If the parameter b_i is chosen as $b_i = 0$, then the event-triggered mechanism will be transformed into the traditional time-triggered case when the system tends to the steady state, that is, $\lim_{t\to\infty} q_i(t) = 0$, which may lead to the Zeno behavior. To avoid this case, a method of introducing an extra positive constant b_i into the event-triggered mechanism will be proposed to eliminate the Zeno behavior, which is inspired by [19].

Combined with (6), the control input $u_i(t)$ is rewritten as follows:

$$\begin{aligned} u_i(t) = K &\sum_{j\in N_i\cap\mathcal{F}} a_{ij}\Big(\hat{x}_i\big(t_k^i\big) - \hat{x}_j\Big(t_{k'}^j\Big)\Big) \\ &+ K \sum_{j\in N_i\cap\Omega} a_{ij}\big(\hat{x}_i\big(t_k^i\big) - x_j(t)\big), \quad i \in \mathcal{F}. \end{aligned} \tag{7}$$

For convenience to the presentation, let $\tilde{x}_i(k) \triangleq \hat{x}_i(t_k^i)$ and denote

$$\begin{aligned} x_f(t) &= \big[x_1^T(t), \ldots, x_M^T(t)\big]^T \\ x_l(t) &= \big[x_{M+1}^T(t), \ldots, x_N^T(t)\big]^T \\ \hat{x}_f(t) &= \big[\hat{x}_1^T(t), \ldots, \hat{x}_M^T(t)\big]^T \\ \tilde{x}_f(t) &= \big[\tilde{x}_1^T(t), \ldots, \tilde{x}_M^T(t)\big]^T \\ m_f(t) &= \big[m_1^T(t), \ldots, m_M^T(t)\big]^T. \end{aligned} \tag{8}$$

Then, according to (1), (3), (7), and (8), we can obtain the following closed-loop system:

$$\begin{aligned} \dot{x}_f(t) &= (I_M \otimes A)x_f(t) + (L_1 \otimes BK)\tilde{x}_f(t) + (L_2 \otimes BK)x_l(t) \\ \dot{\hat{x}}_f(t) &= (I_M \otimes A)\hat{x}_f(t) + (L_1 \otimes BK)\tilde{x}_f(t) + (L_2 \otimes BK)x_l(t) \\ &\quad + (I_M \otimes GC)\big(x_f(t) - \hat{x}_f(t)\big). \end{aligned}$$

The dynamic of leaders according to (2) and (8) is rewritten as follows:

$$\dot{x}_l(t) = (I_{N-M} \otimes A)x_l(t).$$

Defining estimation error as $e_f(t) = x_f(t) - \hat{x}_f(t)$, then

$$\dot{e}_f(t) = (I_M \otimes (A - GC))e_f(t) \tag{9}$$

where $e_f(t) = [e_1^T(t), \ldots, e_M^T(t)]^T$ with $e_i(t) = x_i(t) - \hat{x}_i(t)$.

In order to study the containment control problem, define $\bar{x}_f(t) = x_f(t) + (L_1^{-1}L_2 \otimes I_{N-M})x_l(t)$. Then, we have

$$\begin{aligned} \dot{\bar{x}}_f(t) &= (I_M \otimes A + L_1 \otimes BK)\bar{x}_f(t) - (L_1 \otimes BK)e_f(t) \\ &\quad - (I_M \otimes BK)m_f(t). \end{aligned} \tag{10}$$

Defining $z(t) = [\bar{x}_f^T(t), e_f^T(t)]^T$, the following compact form can be obtained:

$$\begin{aligned} \dot{z}(t) &= \begin{bmatrix} I_M \otimes A + L_1 \otimes BK & -L_1 \otimes BK \\ 0 & I_M \otimes (A - GC) \end{bmatrix} z(t) \\ &\quad + \begin{bmatrix} -I_M \otimes BK \\ 0 \end{bmatrix} m_f(t). \end{aligned}$$

In order to analyze the stability of the closed-loop system only using the eigenvalues of the Laplacian matrix, denoting $\bar{z}(t) = (S^T \otimes I_{2n})z(t)$, we can obtain

$$\dot{\bar{z}}(t) = \bar{A}\bar{z}(t) + \bar{E}m_f(t) \tag{11}$$

where

$$\bar{A} = \begin{bmatrix} I_M \otimes A + \Lambda \otimes BK & -\Lambda \otimes BK \\ 0 & I_M \otimes (A - GC) \end{bmatrix}$$

and $\bar{E} = \begin{bmatrix} -S^T \otimes BK \\ 0 \end{bmatrix}$.

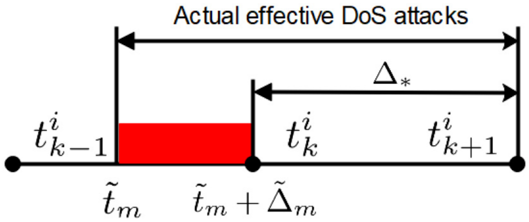

Fig. 2. Schematics of DoS attacks and event-triggered instants.

C. Problem Presentation Under DoS Attacks

The schematics of DoS attacks and event-triggered instants are shown in Fig. 2. In this article, we assume the DoS attacks sequence as $\{\tilde{t}_m\}_{m\in\mathbb{N}}$, where $\tilde{t}_m$ is the start instant of DoS attacks. The mth DoS attacks interval is defined as $[\tilde{t}_m, \tilde{t}_m + \tilde{\Delta}_m)$ with $\tilde{\Delta}_m$ being the duration of DoS attacks and $\tilde{t}_{m+1} > \tilde{t}_m + \tilde{\Delta}_m$. The intervals of DoS attacks are same for all agents. The union of DoS attacks interval $[t_0, t)$ is given by

$$\Xi_a(t_0, t) = \left\{\bigcup_{m\in\mathbb{N}}\left[\tilde{t}_m, \tilde{t}_m + \tilde{\Delta}_m\right)\right\}\bigcap[t_0, t]. \tag{12}$$

The time intervals without DoS attacks are given by

$$\Xi_s(t_0, t) = [t_0, t] \setminus \Xi_a(t_0, t). \tag{13}$$

Similar to [39], the upper bound of two consecutive interevent time is defined as Δ_*, that is, $\Delta_* = \sup_{i,k}\{t^i_{k+1} - t^i_k\}$. It is noted that the event-triggered mechanism does not work in $[0, \Delta_*)$, that is, the control input does not update in $[0, \Delta_*)$. In addition, during the time of DoS attacks, the control input also does not update, which is similar to the former case. Therefore, the duration of "actual effective" DoS attacks include the actual DoS attacks duration and $[0, \Delta_*)$, which is longer than the actual DoS attacks duration. Then, the total time intervals of DoS attacks are obtained as

$$\tilde{\Xi}_a(t_0, t) = \left\{\bigcup_{m\in\mathbb{N}}\left[\tilde{t}_m, \tilde{t}_m + \tilde{\Delta}_m + \Delta_*\right)\right\}\bigcap[t_0, t]. \tag{14}$$

Obviously, the total valid communication time intervals are represented as

$$\tilde{\Xi}_s(t_0, t) = [t_0, t] \setminus \tilde{\Xi}_a(t_0, t). \tag{15}$$

Assumption 2: Denoting $n_a(t_0, t)$ as the amount of DoS attacks occurring in $[t_0, t)$, then the frequency of DoS attacks $F_a(t_0, t) > 0$ over $[t_0, t)$ satisfies

$$n_a(t_0, t) \le F_a(t_0, t)(t - t_0). \tag{16}$$

Assumption 3: Defining $|\Xi_a(t_0, t)|$ as the total actual time intervals in the presence of DoS attacks in $[t_0, t)$, then the total durations of DoS attacks satisfy the following condition:

$$|\Xi_a(t_0, t)| \le \Xi_0 + \frac{t - t_0}{\tau_a} \tag{17}$$

where $\tau_a > 1$ and Ξ_0 are positive constants to be determined.

Remark 4: Assumptions 2 and 3 have been used in [45] and [51]. In Assumption 2, it specifies the frequency of DoS attacks, and its upper bound will be determined later. Assumption 3 implies that the total duration of DoS attacks is constrained by the certain fraction of time. Besides, the role of Ξ_0 is to consider the case that DoS attacks exist at the start time.

Remark 5: It is noted that there are some differences between the traditional packet dropouts phenomenon and DoS attacks. The number of packet dropouts usually belongs to an integer set, and the number of continuous packet dropouts is less than a small number, while DoS attacks may last for a long period of time. For example, when the communication failure frequency is less than a certain value, the considered DoS attacks may include the case in which the network is blocked for a certain period of time. However, this case usually does not fall into the category of packet dropouts due to the fact that the continuous packet dropouts are less than a small number.

Problem 1: For the closed-loop system (11) subjected to DoS attacks, the goal in this article is to develop a resilient event-triggered containment controller based on the designed observer to guarantee

$$\lim_{t\to\infty} \bar{z}(t) = 0. \tag{18}$$

Remark 6: According to the definition of $\bar{z}(t)$ and Lemma 1, (18) implies the following equality holding:

$$\lim_{t\to\infty}\left[x_f(t) - \left(-L_1^{-1}L_2 \otimes I_{N-M}\right)x_l(t)\right] = 0.$$

Invoking Definition 1, we know that the containment control objective for MAS (1) can be achieved.

III. Stability Analysis

The stability conditions that ensuring MASs can achieve the containment control objective with the consideration of DoS attacks is provided in this section.

It is noted that the controller and observer will fail when the system suffers from DoS attacks, that is, the system will be transformed into an open-loop system. Then, according to the DoS attacks model proposed in the previous section, we can obtain

$$\begin{cases} \dot{\bar{z}}(t) = \bar{A}\bar{z}(t) + \bar{E}m_f(t), & t \in \tilde{\Xi}_s(t_0, \infty) \\ \dot{\bar{z}}(t) = \breve{A}\bar{z}(t), & t \in \tilde{\Xi}_a(t_0, \infty) \end{cases} \tag{19}$$

where $\breve{A} = \begin{bmatrix} I_M \otimes A & 0 \\ 0 & I_M \otimes A \end{bmatrix}$. $\tilde{\Xi}_a(t_0, \infty)$ and $\tilde{\Xi}_s(t_0, \infty)$ are the total time intervals with and without DoS attacks defined by (14) and (15), respectively.

The following theorem is the main result which guarantees MAS (1) achieving the states containment.

Theorem 1: For known controller gain K, observer gain G, positive constants a_1, a_2 and $\eta_1^* \in (0, a_1)$, Problem 1 can be solved if there exist positive-definite matrix $\mathcal{P} = I_M \otimes U$ with $U = \text{diag}\{P, Q\}$ and $P, Q \in \mathbb{R}^{n\times n}$, constants $\gamma = \gamma_1 + \gamma_2$ with $\gamma_1 \ge \max_{i=1,\ldots,M}\{\theta_i\}$, $\gamma_2 > 0$, $b_i \le \varrho$, and Δ_* such that the following matrix inequalities hold:

$$\Pi + a_1\mathcal{P} < 0 \tag{20}$$

$$\mathcal{P}\breve{A} + \breve{A}^T\mathcal{P} - a_2\mathcal{P} < 0 \tag{21}$$

and DoS attacks satisfy

$$F_a(t_0, t) - \frac{\eta_1^*}{(a_1 + a_2)\Delta_*} \leq 0 \tag{22}$$

$$-\tau_a + \frac{a_1 + a_2}{a_1 - \eta_1^*} < 0 \tag{23}$$

where

$$\Pi = \mathcal{P}\bar{A} + \bar{A}^T\mathcal{P} + \mathcal{P}\bar{E}\bar{E}^T\mathcal{P} + I_\delta^T(I_M \otimes C)^T\Upsilon^T\Upsilon(I_M \otimes C)I_\delta$$

$$\varrho = \frac{1}{\sigma}ln\left(1 + \frac{c}{\|L_1\|}\right), \quad \sigma = \|\bar{A}\| + \|\bar{E}\|\|L_1\|, \quad c = \frac{\gamma_2\|C\|}{M}$$

with $\Upsilon = \text{diag}\{\gamma, \ldots, \gamma\}$ and $I_\delta = [0_M, I_M]$.

Proof: The proof is divided into two parts: 1) the stability analysis of the closed-loop system and 2) eliminating the Zeno behavior.

Part-I (Stability Analysis): Define the Lyapunov function as $V(t) = \bar{z}^T(t)\mathcal{P}\bar{z}(t)$. If there are no DoS attacks in the system, that is, $t \in \tilde{\Xi}_s(t_0, \infty)$, the derivative of $V(t)$ is calculated as

$$\dot{V}(t) = 2\bar{z}^T(t)\mathcal{P}\dot{\bar{z}} = 2\bar{z}^T(t)\mathcal{P}\big(\bar{A}\bar{z}(t) + \bar{E}m_f(t)\big). \tag{24}$$

Using Young's inequality, one has

$$2\bar{z}^T(t)\mathcal{P}\bar{E}m_f(t) \leq \bar{z}^T(t)\mathcal{P}\bar{E}\bar{E}^T\mathcal{P}\bar{z}(t) + m_f^T(t)m_f(t). \tag{25}$$

According to the event-triggered condition, the following inequality can be obtained:

$$m_f^T(t)m_f(t) \leq \bar{z}^T(t)I_\delta^T(I_M \otimes C)^T\Upsilon^T\Upsilon(I_M \otimes C)I_\delta\bar{z}(t). \tag{26}$$

It is worth pointing that inequality (26) will be guaranteed in the following proof of estimating the Zeno behavior. Then, one can obtain the following inequality from (24)–(26):

$$\begin{aligned}\dot{V}(t) &\leq 2\bar{z}^T(t)\mathcal{P}\bar{A}\bar{z}(t) + \bar{z}^T(t)\mathcal{P}\bar{E}\bar{E}^T\mathcal{P}\bar{z}(t) + m_f^T(t)m_f(t)\\ &\leq \bar{z}^T(t)\Pi\bar{z}(t).\end{aligned}$$

According to (20), one has

$$\dot{V}(t) \leq -a_1V(t). \tag{27}$$

If there are DoS attacks in the system, that is, $t \in \tilde{\Xi}_a(t_0, \infty)$. Similar to the case without DoS attacks in the system, one obtains the following inequality from (21):

$$\dot{V}(t) = \bar{z}^T(t)\Big(\mathcal{P}\breve{A} + \breve{A}^T\mathcal{P}\Big)\bar{z}(t) \leq a_2V(t). \tag{28}$$

Define $[\tilde{t}_{m-1} + \bar{\Delta}_{m-1}, \tilde{t}_m) \triangleq T_1$ and $[\tilde{t}_m, \tilde{t}_m + \bar{\Delta}_m) \triangleq T_2$. Then, according to [48], one can follow from (27) and (28) that:

$$V(t) \leq \begin{cases} e^{-a_1(t-\tilde{t}_{m-1}-\bar{\Delta}_{m-1})}V\big(\tilde{t}_{m-1} + \bar{\Delta}_{m-1}\big), & t \in T_1\\ e^{a_2(t-\tilde{t}_m)}V\big(\tilde{t}_m\big), & t \in T_2.\end{cases} \tag{29}$$

Note that there exist two cases for any time t, that is, $t \in T_1$ or $t \in T_2$, which is illustrated in Fig. 3. If $t \in T_1$, one has

$$\begin{aligned}V(t) &\leq e^{-a_1(t-\tilde{t}_{m-1}-\bar{\Delta}_{m-1})}V\big(\tilde{t}_{m-1} + \bar{\Delta}_{m-1}\big)\\ &\leq e^{-a_1(t-\tilde{t}_{m-1}-\bar{\Delta}_{m-1})}e^{a_2(\tilde{t}_{m-1}+\bar{\Delta}_{m-1}-\tilde{t}_{m-1})}V\big(\tilde{t}_{m-1}\big)\\ &\leq e^{-a_1(t-\tilde{t}_{m-1}-\bar{\Delta}_{m-1})}e^{a_2(\tilde{t}_{m-1}+\bar{\Delta}_{m-1}-\tilde{t}_{m-1})}\\ &\qquad e^{-a_1(\tilde{t}_{m-1}-\tilde{t}_{m-2}-\bar{\Delta}_{m-2})}V\big(\tilde{t}_{m-2} + \bar{\Delta}_{m-2}\big)\\ &\leq \ldots\\ &\leq e^{-a_1\left|\tilde{\Xi}_s(t_0,t)\right|}e^{a_2\left|\tilde{\Xi}_a(t_0,t)\right|}V(t_0)\end{aligned} \tag{30}$$

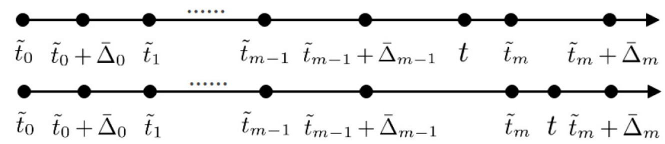

Fig. 3. Situation of DoS attacks.

in which $|\tilde{\Xi}_a(t_0, t)| = \tilde{t}_{m-1} + \bar{\Delta}_{m-1} - \tilde{t}_{m-1} + \tilde{t}_{m-2} + \bar{\Delta}_{m-2} - \tilde{t}_{m-2} + \ldots + \tilde{t}_0 + \bar{\Delta}_0 - \tilde{t}_0$ and $|\tilde{\Xi}_s(t_0, t)| = t - \tilde{t}_{m-1} - \bar{\Delta}_{m-1} + \tilde{t}_{m-1} - \tilde{t}_{m-2} - \bar{\Delta}_{m-2} + \ldots + \tilde{t}_1 - \tilde{t}_0 - \bar{\Delta}_0$.

If $t \in T_2$, one has

$$\begin{aligned}V(t) &\leq e^{a_2(t-\tilde{t}_m)}V\big(\tilde{t}_m\big)\\ &\leq e^{a_2(t-\tilde{t}_m)}e^{-a_1(\tilde{t}_m-\tilde{t}_{m-1}-\bar{\Delta}_{m-1})}V\big(\tilde{t}_{m-1} + \bar{\Delta}_{m-1}\big)\\ &\leq \ldots\\ &\leq e^{-a_1\left|\tilde{\Xi}_s(t_0,t)\right|}e^{a_2\left|\tilde{\Xi}_a(t_0,t)\right|}V(t_0).\end{aligned} \tag{31}$$

It is not difficult to obtain that $|\tilde{\Xi}_s(t_0, t)| = t - t_0 - |\tilde{\Xi}_a(t_0, t)|$ and $|\tilde{\Xi}_a(t_0, t)| \leq |\Xi_a(t_0, t)| + (1 + n_a(t_0, t))\Delta_*$ for all $t \geq t_0$. Combining (30) with (31), one can obtain

$$\begin{aligned}V(t) &\leq e^{-a_1\left|\tilde{\Xi}_s(t_0,t)\right|}e^{a_2\left|\tilde{\Xi}_a(t_0,t)\right|}V(t_0)\\ &= e^{-a_1\left(t-t_0-\left|\tilde{\Xi}_a(t_0,t)\right|\right)}e^{a_2\left|\tilde{\Xi}_a(t_0,t)\right|}V(t_0)\\ &\leq e^{-a_1(t-t_0)+(a_1+a_2)\frac{t-t_0}{\tau_a}}\\ &\quad\times e^{(a_1+a_2)(\Xi_0+(1+n_a(t_0,t))\Delta_*)}V(t_0)\\ &= e^{(a_1+a_2)(\Xi_0+\Delta_*)}e^{\left(-a_1+\frac{a_1+a_2}{\tau_a}\right)(t-t_0)}\\ &\quad\times e^{n_a(t_0,t)[(a_1+a_2)\Delta_*]}V(t_0).\end{aligned}$$

According to (22), (23), and Assumption 2, one can obtain $([n_a(t_0, t)]/(t - t_0)) \leq F_a(t_0, t) \leq (\eta_1^*/[(a_1 + a_2)\Delta_*])$ and $\tau_a > [(a_1 + a_2)/(a_1 - \eta_1^*)]$, then

$$\begin{aligned}V(t) &\leq e^{(a_1+a_2)(\Xi_0+\Delta_*)}e^{\left(-a_1+\frac{a_1+a_2}{\tau_a}\right)(t-t_0)}e^{\eta_1^*(t-t_0)}V(t_0)\\ &= e^{(a_1+a_2)(\Xi_0+\Delta_*)}e^{\left(-a_1+\frac{a_1+a_2}{\tau_a}+\eta_1^*\right)(t-t_0)}V(t_0)\\ &\leq e^{(a_1+a_2)(\Xi_0+\Delta_*)}e^{-\eta_1(t-t_0)}V(t_0)\end{aligned} \tag{32}$$

where $\eta_1 = a_1 - [(a_1 + a_2)/\tau_a] - \eta_1^* > 0$.

Obviously, it can be seen from (32) that $\lim_{t\to\infty}\bar{z}(t) = 0$. As mentioned in Remark 6, the states containment can be achieved.

Part-II (Eliminating the Zeno Behavior): In the following, the lower bound of the interevent time will be given, which can eliminate the Zeno behavior effectively. Let $\mathcal{W}_1$ be the agent sets in which the event-triggered instant is determined by τ_k^i, and $\mathcal{W}_2$ be the agent sets where the event-triggered instant is determined by b_i. Then, one can obtain that $\mathcal{W}_1 \cup \mathcal{W}_2 = \mathcal{F}$ and $\mathcal{W}_1 \cap \mathcal{W}_2 = \emptyset$. To ensure that inequality (26) holds, we can select $\gamma = \gamma_1 + \gamma_2$ with $\gamma_1 > 0$ and $\gamma_2 > 0$ such that

$$\sum_{i\in\mathcal{W}_1}\|m_i(t)\| \le \gamma_1 \sum_{i\in\mathcal{W}_1}\|Ce_i(t)\| \le \gamma_1 \sum_{i=1}^{M}\|Ce_i(t)\| \quad (33)$$

$$\sum_{i\in\mathcal{W}_2}\|m_i(t)\| \le \gamma_2 \sum_{i\in\mathcal{W}_2}\|Ce_i(t)\| \le \gamma_2 \sum_{i=1}^{M}\|Ce_i(t)\|. \quad (34)$$

A sufficient condition to ensure (33) holding is that $\gamma_1 \ge \max_{i=1,\dots,M}\{\theta_i\}$ for agents in $\mathcal{W}_1$. In addition, for the agents in $\mathcal{W}_2$, a sufficient condition for (34) is $\|m_i(t)\| \le \sum_{i=1}^{M}(\gamma_2\|C\|/M)\|e_i(t)\| \le (\gamma_2\|C\|/M)\|e(t)\| \le (\gamma_2\|C\|/M)\|\bar{z}(t)\|$. Letting $c = (\gamma_2\|C\|/M)$, then $\|m_i(t)\| \le c\|\bar{z}(t)\|$. If let b_i be the lower bound that $([\|m_i(t)\|]/\|\bar{z}(t)\|)$ evolves from 0 to c for any agent in $\mathcal{W}_2$, then $t_{k+1}^i = t_k^i + b_i$ can ensure (34) holding. Next, to find such lower time bound b_i, one can estimate the time derivative of $(\|m_i(t)\|/[\|\bar{z}(t)\|])$

$$\begin{aligned}
\frac{d}{dt}\frac{\|m_i(t)\|}{\|\bar{z}(t)\|} &= \frac{m_i^T(t)\dot{m}_i(t)}{\|m_i(t)\|\|\bar{z}(t)\|} - \frac{\|m_i(t)\|\bar{z}^T(t)\dot{\bar{z}}(t)}{\|\bar{z}(t)\|^3}\\
&\le \frac{\|\dot{m}_i(t)\|}{\|\bar{z}(t)\|} + \frac{\|m_i(t)\|}{\|\bar{z}(t)\|}\frac{\|\dot{\bar{z}}(t)\|}{\|\bar{z}(t)\|}\\
&\le \left(\|L_1\| + \frac{\|m_i(t)\|}{\|\bar{z}(t)\|}\right)\frac{\|\dot{\bar{z}}(t)\|}{\|\bar{z}(t)\|}\\
&= \left(\|L_1\| + \frac{\|m_i(t)\|}{\|\bar{z}(t)\|}\right)\frac{\|\bar{A}\bar{z}(t) + \bar{E}m_f(t)\|}{\|\bar{z}(t)\|}\\
&\le \left(\|L_1\| + \frac{\|m_i(t)\|}{\|\bar{z}(t)\|}\right)(\|\bar{A}\| + \|\bar{E}\|\|L_1\|)\\
&= \sigma\left(\|L_1\| + \frac{\|m_i(t)\|}{\|\bar{z}(t)\|}\right).
\end{aligned}$$

Then, $(\|m_i(t)\|/\|\bar{z}(t)\|)$ satisfies the condition $(\|m_i(t)\|/\|\bar{z}(t)\|) < f^*$ with f^* being the solution of $\dot{f}^* = \sigma(\|L_1\| + f^*)$. Besides, ϱ is an upper bound of $(\|m_i(t)\|/\|\bar{z}(t)\|)$ evolved from 0 to c. So, $b_i < \varrho$ guarantees (34) holding for the agents in $\mathcal{W}_2$. Therefore, according to (33), (34), and the definition of Υ in Theorem 1, inequality (26) holds. ■

Remark 7: In (30) and (31), when $t \in T_1$, the inequality $V(t) \le e^{-a_1(t-\tilde{t}_{m-1}-\tilde{\Delta}_{m-1})}V(\tilde{t}_{m-1}+\tilde{\Delta}_{m-1})$ holds, and when $t \in T_2$, the inequality $V(t) \le e^{a_2(t-\tilde{t}_m)}V(\tilde{t}_m)$ holds. Then, keep on iterating and we can obtain $V(t) \le e^{-a_1|\tilde{\Xi}_s(t_0,t)|}e^{a_2|\tilde{\Xi}_a(t_0,t)|}V(t_0)$ with the help of definitions of $|\tilde{\Xi}_a(t_0,t)|$ and $|\tilde{\Xi}_s(t_0,t)|$.

IV. Distributed Controller and Observer Design

This section aims to design the observer-based controller to solve Problem 1, which can be implemented by Theorem 2 using an improved separation method.

Theorem 2: For given positive constants a_1, a_2, and $\eta_1^* \in (0, a_1)$, Problem 1 can be solved, if there exist matrices $Q > 0$, $W > 0$, Z, Y, constants $\gamma = \gamma_1 + \gamma_2$ with $\gamma_1 \ge \max_{i=1,\dots,M}\{\theta_i\}$, $\gamma_2 > 0$, $\delta_1 > 0$, $\delta_2 > 0$, $b_i \le \varrho$, and Δ_* such that the following LMIs are satisfied for $j = 1, 2$, $i = 1, \dots, M$:

$$\Psi^i \triangleq \begin{bmatrix} \Xi_1^i & BZ & -\lambda_i BZ\\ * & -\delta_2 W & 0\\ * & * & -\delta_1 W\end{bmatrix} < 0 \quad (35)$$

$$\begin{bmatrix} \Xi_2 & 0\\ * & -\delta_1 W\end{bmatrix} < 0 \quad (36)$$

$$\begin{bmatrix} \Xi_4^j & I\\ * & -\delta_1^{-1} W\end{bmatrix} < 0 \quad (37)$$

$$\delta_2 I - W < 0 \quad (38)$$

and DoS attack satisfy

$$F_a(t_0,t) - \frac{\eta_1^*}{(a_1+a_2)\Delta_*} \le 0 \quad (39)$$

$$-\tau_a + \frac{a_1+a_2}{a_1-\eta_1^*} < 0 \quad (40)$$

where

$$\begin{aligned}
&\Xi_1^i = AW + WA^T + \lambda_i BZ + \lambda_i Z^T B^T + a_1 W\\
&\Xi_2 = AW + WA^T - a_2 W,\ \Xi_4^1 = QA + A^T Q - a_2 Q\\
&\Xi_4^2 = QA + A^T Q - YC - (YC)^T + \gamma^2 C^T C + a_1 Q\\
&\varrho = \frac{1}{\sigma}\ln\left(1 + \frac{c}{\|L_1\|}\right),\ \sigma = \|\bar{A}\| + \|\bar{E}\|\|L_1\|,\ c = \frac{\gamma_2\|C\|}{M}.
\end{aligned}$$

Furthermore, the observer and the controller gains are calculated by $G = Q^{-1}Y$ and $K = ZW^{-1}$, respectively.

Proof: Notice that the terms in (20) and (21) are block-diagonal matrices. Facilitating to deal with them, we, respectively, rewrite (20) and (21) as

$$U\bar{A}_i + \bar{A}_i^T U + U\tilde{E}_i\tilde{E}_i^T U + \gamma^2 D^T C^T C D + a_1 U < 0 \quad (41)$$

$$U\tilde{A}^T + \tilde{A}U - a_2 U < 0 \quad (42)$$

where

$$\bar{A}_i = \begin{bmatrix} A + \lambda_i BK & -\lambda_i BK\\ 0 & A - GC\end{bmatrix},\quad D = [0, I]$$

$$\tilde{E}_i = \begin{bmatrix} BK\\ 0\end{bmatrix},\quad \tilde{A} = \begin{bmatrix} A & 0\\ 0 & A\end{bmatrix}$$

and the following transformation is used:

$$\begin{aligned}
\bar{E}\bar{E}^T &= \begin{bmatrix} S^T S \otimes BK(BK)^T & 0\\ 0 & 0\end{bmatrix}\\
&= \begin{bmatrix} I_M \otimes BK(BK)^T & 0\\ 0 & 0\end{bmatrix} = \tilde{E}\tilde{E}^T
\end{aligned}$$

where $\tilde{E} = \begin{bmatrix} I_M \otimes BK\\ 0\end{bmatrix}$.

Then, taking $\bar{A}_i$, $\tilde{A}$, $\tilde{E}_i$, and $U = \mathrm{diag}\{P, Q\}$ into inequalities (41) and (42), one can obtain the following matrix inequalities:

$$\Gamma_1 = \begin{bmatrix} \Xi_5^i & -\lambda_i PBK\\ * & \Xi_6\end{bmatrix} < 0,\quad \Gamma_2 = \begin{bmatrix} \Xi_7 & 0\\ * & \Xi_8\end{bmatrix} < 0$$

where $\Xi_5^i = PA + A^T P + \lambda_i PBK + \lambda_i K^T B^T P + PBKK^T B^T P + a_1 P$, $\Xi_6 = QA + A^T Q - QGC - C^T G^T Q + \gamma^2 C^T C + a_1 Q$ and $\Xi_7 = PA + A^T P - a_2 P$, $\Xi_8 = QA + A^T Q - a_2 Q$.

It is noted that Γ_1 and Γ_2 can be rewritten as $\Gamma_1 = \Upsilon_1 + \Upsilon_2$ and $\Gamma_2 = \Upsilon_3 + \Upsilon_4$, where

$$\Upsilon_1 = \begin{bmatrix} \Xi_5^i & -\lambda_i PBK\\ * & -\delta_1 P\end{bmatrix},\quad \Upsilon_2 = \begin{bmatrix} 0 & 0\\ * & \Xi_6 + \delta_1 P\end{bmatrix}$$

$$\Upsilon_3 = \begin{bmatrix} \Xi_7 & 0\\ * & -\delta_1 P\end{bmatrix},\quad \Upsilon_4 = \begin{bmatrix} 0 & 0\\ * & \Xi_8 + \delta_1 P\end{bmatrix}.$$

Obviously, it can be seen that $\Upsilon_1 < 0$, $\Xi_6 + \delta_1 P < 0$, $\Upsilon_3 < 0$, and $\Xi_8 + \delta_1 P < 0$ ensure $\Gamma_1 < 0$ and $\Gamma_2 < 0$, that is, matrix inequalities (41) and (42) hold. It is obvious that Υ_1, $\Xi_6+\delta_1 P$, Υ_3, and $\Xi_8+\delta_1 P$ are the nonlinear matrix inequalities. In order to transform them into LMIs, the following arrangement will be done.

Using the Schur complement lemma and multiplying left and right with the matrix diag$\{P^{-1}, P^{-1}, P^{-1}\}$ for Υ_1 leads to

$$\Upsilon_5 = \begin{bmatrix} \Xi_8^i & BKP^{-1} & -\lambda_i BKP^{-1} \\ * & -P^{-1}P^{-1} & 0 \\ * & * & -\delta_1 P^{-1} \end{bmatrix} < 0$$

where $\Xi_8^i = AP^{-1}+P^{-1}A^T+\lambda_i BKP^{-1}+\lambda_i P^{-1}K^T B^T+a_1 P^{-1}$.

Further, Υ_5 can be rewritten as $\Upsilon_5 = \Upsilon_6 + \Upsilon_7$, where

$$\Upsilon_6 = \begin{bmatrix} \Xi_8^i & BKP^{-1} & -\lambda_i BKP^{-1} \\ * & -\delta_2 P^{-1} & 0 \\ * & * & -\delta_1 P^{-1} \end{bmatrix}$$

$$\Upsilon_7 = \begin{bmatrix} 0 & 0 & 0 \\ * & \delta_2 P^{-1} - P^{-1}P^{-1} & 0 \\ * & * & 0 \end{bmatrix}.$$

$\Upsilon_6 < 0$ and $\delta_2 P^{-1} - P^{-1}P^{-1} < 0$ can ensure $\Upsilon_5 < 0$ holding. In addition, it is obvious that $\delta_2 P^{-1} - P^{-1}P^{-1} < 0$ is equivalent to $\delta_2 I - P^{-1} < 0$.

Multiplying left and right with the matrix diag$\{P^{-1}, P^{-1}\}$ for Υ_3 leads to

$$\begin{bmatrix} \Xi_9 & 0 \\ * & -\delta_1 P^{-1} \end{bmatrix} < 0$$

where $\Xi_9 = AP^{-1} + P^{-1}A^T - a_2 P^{-1}$.

Then, defining $W = P^{-1}$ and $Z = KW$ result in (35) and (36). Besides, LMIs (37) can be obtained by using the Schur complement lemma and defining $Y = QG$ for $\Xi_6 + \delta_1 P$ and $\Xi_8 + \delta_1 P$. ∎

Remark 8: Different from the observer-based controller design method in [36] and [44], in which the control input matrix is assumed to be full-column rank, which is introduced to guarantee that the SVD and orthogonal basis method can be used to obtain the observer and controller gains. By using the improved separation method, the full-column rank constraint of the control input matrix is removed, which implies the proposed observer-based controller design method is less conservative than those in [36] and [44].

It is noted that Theorem 2 depends on all eigenvalues of the Laplacian matrix. Thus, in order to reduce the controller design burden and use less information of the Laplacian matrix, the following corollary is given.

Corollary 1: For given positive constants a_1, a_2, and $\eta_1^* \in (0, a_1)$, Problem 1 can be solved, if there exist matrices $Q > 0$, $W > 0$, Z, Y, constants $\gamma = \gamma_1+\gamma_2$ with $\gamma_1 \geq \max_{i=1,\dots,M}\{\theta_i\}$, $\gamma_2 > 0$, $\delta_1 > 0$, $\delta_2 > 0$, $b_i \leq \varrho$, $\tau_i > 0 (i = 1, 2, 3)$, and Δ_* such that the following LMIs are satisfied for $j = 1, 2$:

$$\begin{bmatrix} \Xi_1 & BZ & -\lambda_1 BZ \\ * & -\delta_2 W + \bar{\lambda}\tau_1 I & 0 \\ * & * & -\delta_1 W + \bar{\lambda}\tau_2 I \end{bmatrix} < 0 \quad (43)$$

$$\begin{bmatrix} -BZ - Z^T B^T - \tau_0 I & 0 & -BZ \\ * & -\tau_1 I & 0 \\ * & * & -\tau_2 I \end{bmatrix} < 0 \quad (44)$$

$$\begin{bmatrix} \Xi_2 & 0 \\ * & -\delta_1 W \end{bmatrix} < 0 \quad (45)$$

$$\begin{bmatrix} \Xi_4^j & I \\ * & -\delta_1^{-1} W \end{bmatrix} < 0 \quad (46)$$

$$\delta_2 I - W < 0 \quad (47)$$

and DoS attack satisfy

$$F_a(t_0, t) - \frac{\eta_1^*}{(a_1 + a_2)\Delta_*} \leq 0 \quad (48)$$

$$-\tau_a + \frac{a_1 + a_2}{a_1 - \eta_1^*} < 0 \quad (49)$$

where

$$\Xi_1 = AW + WA^T + \lambda_1 BZ + \lambda_1 Z^T B^T + a_1 W + \bar{\lambda}\tau_0 I$$
$$\Xi_2 = AW + WA^T - a_2 W,\ \Xi_4^1 = QA + A^T Q - a_2 Q$$
$$\Xi_4^2 = QA + A^T Q - YC - (YC)^T + \gamma^2 C^T C + a_1 Q$$
$$\varrho = \frac{1}{\sigma} ln\left(1 + \frac{c}{\|L_1\|}\right),\quad \sigma = \|\bar{A}\| + \|\bar{E}\| \|L_1\|$$
$$c = \frac{\gamma_2 \|C\|}{M},\quad \bar{\lambda} = \lambda_1 - \lambda_M.$$

Furthermore, the observer and controller gains are calculated by $G = Q^{-1}Y$ and $K = ZW^{-1}$, respectively.

Proof: The proof is divided into two cases: 1) $\lambda_1 > \lambda_M$ and 2) $\lambda_1 = \lambda_M$. When $\lambda_1 > \lambda_M$, (35) can be rewritten as

$$\begin{aligned}\Psi^i = \Psi^1 &+ (\lambda_i - \lambda_1)\begin{bmatrix} BZ + Z^T B^T + \tau_0 I & 0 & BZ \\ * & \tau_1 I & 0 \\ * & * & \tau_2 I \end{bmatrix} \\ &+ (\lambda_1 - \lambda_M)\text{diag}\{\tau_0 I, \tau_1 I, \tau_2 I\} \\ &- (\lambda_i - \lambda_M)\text{diag}\{\tau_0 I, \tau_1 I, \tau_2 I\}. \end{aligned} \quad (50)$$

Considering $\lambda_1 \geq \cdots \geq \lambda_M > 0$ and combining with (43) and (44) imply $\Psi^i < 0$ holding, that is, (35) is satisfied. In addition, the same conclusion can be obtained when $\lambda_1 = \lambda_M$, and the rest of the proof is similar to the proof of Theorem 2. ∎

Remark 9: In Theorem 2, the conditions of the controller design depend on all eigenvalues of the Laplacian matrix, which means the computational burden will increase with the number of agents. The conditions given in Corollary 1 are only related to the maximum and minimum eigenvalues of the Laplacian matrix. So we only need to solve five groups of LMIs regardless of how many agents there are, which reduces the computational burden greatly.

V. Simulation

In this section, a practical example of UAVs is presented to testify the effectiveness of the designed observer and event-triggered controller under DoS attacks. The UAV is regarded as a point-mass system in this article, and the dynamics of each UAV can be approximately described as the following second-order system [49], [50]:

$$\dot{x}_i(t) = v_i(t)$$
$$\dot{v}_i(t) = u_i(t)$$
$$y_i(t) = x_i(t)$$

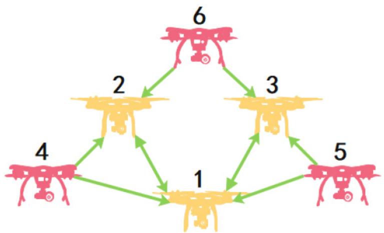

Fig. 4. Network topology $\mathcal{G}$ with six UAVs.

where $x_i(t)$ and $v_i(t)$ represent the position and velocity of the ith UAV, respectively.

Then, the following state space expression can be obtained by considering $z_i(t) = [x_i^T(t), v_i^T(t)]^T$:

$$\dot{z}_i(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} z_i(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i(t)$$

$$y_i(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} z_i(t).$$

Fig. 4 illustrates the communication topology $\mathcal{G}$ with six UAVs, in which the nodes labeled by 4–6 are leaders and the nodes marked by 1–3 are followers. It can be seen from Fig. 4 that the Laplacian matrices are obtained as

$$L_1 = \begin{bmatrix} 4 & -1 & -1 \\ -1 & 3 & 0 \\ -1 & 0 & 3 \end{bmatrix}, \quad L_2 = \begin{bmatrix} -1 & -1 & 0 \\ -1 & 0 & -1 \\ 0 & -1 & -1 \end{bmatrix}.$$

The eigenvalues of L_1 are obtained as 2, 3, and 5, respectively. In this example, the event-triggered thresholds and parameters are chosen as $\theta_1 = 1.5$, $\theta_2 = 2$, $\theta_3 = 2.5$ and $a_1 = 0.5$, $a_2 = 2$, $\eta_1^* = 0.05$, $\gamma_1 = 3$, $\gamma_2 = 5$, $\delta_1 = 50$, $\delta_2 = 15$, respectively. Then, based on (48) and (49), we can obtain $F_a(t_0, t) \leq 0.03$ and $\tau_a \geq 5.56$. Then, letting $\tau_a = 5.6$ and $\Xi_0 = 0.56$ s, we obtain $|\Xi_a(t_0, t)| \leq 2.7$ s. According to Corollary 1, the controller and the observer gains are obtained as $K = [-0.2877, -0.5745]$ and $G = [1.609, 1.5156]^T$, respectively. Besides, we choose $b_i = 0.03$ with $\varrho = 0.037$ and sampling period $T = 0.015$. It is worth noting that there is no feasible solution by using the existing SVD method. Finally, according to Assumptions 2 and 3, the DoS attacks intervals are selected as $[0.015, 0.225) \cup [0.975, 1.275) \cup [2.25, 2.475) \cup [3, 3.3) \cup [3.75, 3.825) \cup [4.95, 5.25) \cup [6.9, 7.2) \cup [7.5, 7.8) \cup [8.7, 9.15) \cup [10.5, 10.725)$.

Selecting the initial states as

$$x_1(t_0) = \begin{bmatrix} -0.15 \\ -0.1 \end{bmatrix}, \quad x_2(t_0) = \begin{bmatrix} 0.2 \\ -0.25 \end{bmatrix}, \quad x_3(t_0) = \begin{bmatrix} -0.3 \\ 0.35 \end{bmatrix}$$

$$x_4(t_0) = \begin{bmatrix} 0.15 \\ 0.14 \end{bmatrix}, \quad x_5(t_0) = \begin{bmatrix} 0.35 \\ 0.3 \end{bmatrix}, \quad x_6(t_0) = \begin{bmatrix} 0.25 \\ 0.24 \end{bmatrix}.$$

The initial states of the observer are chosen as

$$\hat{x}_1(t_0) = \begin{bmatrix} -0.65 \\ -0.7 \end{bmatrix}, \quad \hat{x}_2(t_0) = \begin{bmatrix} 0.7 \\ 0.35 \end{bmatrix}, \quad \hat{x}_3(t_0) = \begin{bmatrix} -0.6 \\ -0.15 \end{bmatrix}.$$

The simulation results in Figs. 5–8 show a comparison between Corollary 1 in this article and the traditional method in [24] without considering the effect of DoS attacks. As can be seen from Figs. 5 and 6 that when DoS attacks occur, the time to achieve the states containment by using the traditional controller designed in [24] is longer than using the resilient controller designed in this article. In addition, from Figs. 7 and 8, the effectiveness illustrated by Corollary 1 can be seen more clearly, that is, figures of the estimation error and containment error of all agents tend to zero rapidly by using Corollary 1. In addition, Table I is provided to further illustrate the effectiveness of the proposed method by calculating the norm-2 of the estimation error of followers during [0, 12 s].

The event-triggered instants for all followers are illustrated in Fig. 9. The number of controller updates using the developed event-triggered mechanism is 280, 262, and 264 (time-triggered communication: 800) for follower 1,

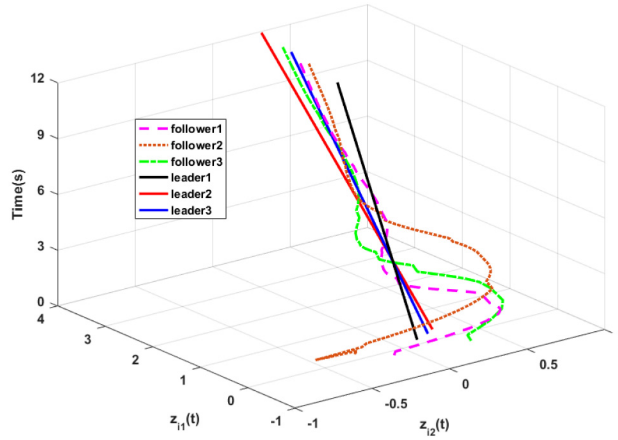

Fig. 5. State trajectories $z_{i1}(t)$ and $z_{i2}(t)$ using Corollary 1 under DoS attacks.

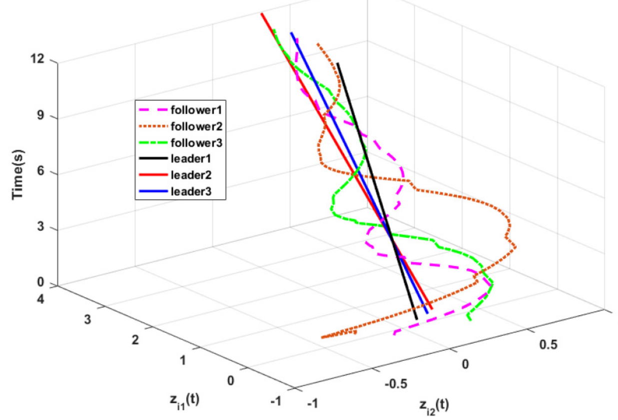

Fig. 6. State trajectories $z_{i1}(t)$ and $z_{i2}(t)$ using the traditional method under DoS attacks.

TABLE I
NORM-2 OF THE ESTIMATION ERROR OF FOLLOWERS

Estimation error	Ours method	Traditional method
$\|e_{11}(t)\|_2$	4.5734	6.2504
$\|e_{12}(t)\|_2$	3.7037	4.9184
$\|e_{21}(t)\|_2$	4.5734	6.2504
$\|e_{22}(t)\|_2$	3.7037	4.9184
$\|e_{31}(t)\|_2$	3.2931	4.5817
$\|e_{32}(t)\|_2$	3.1915	3.9363

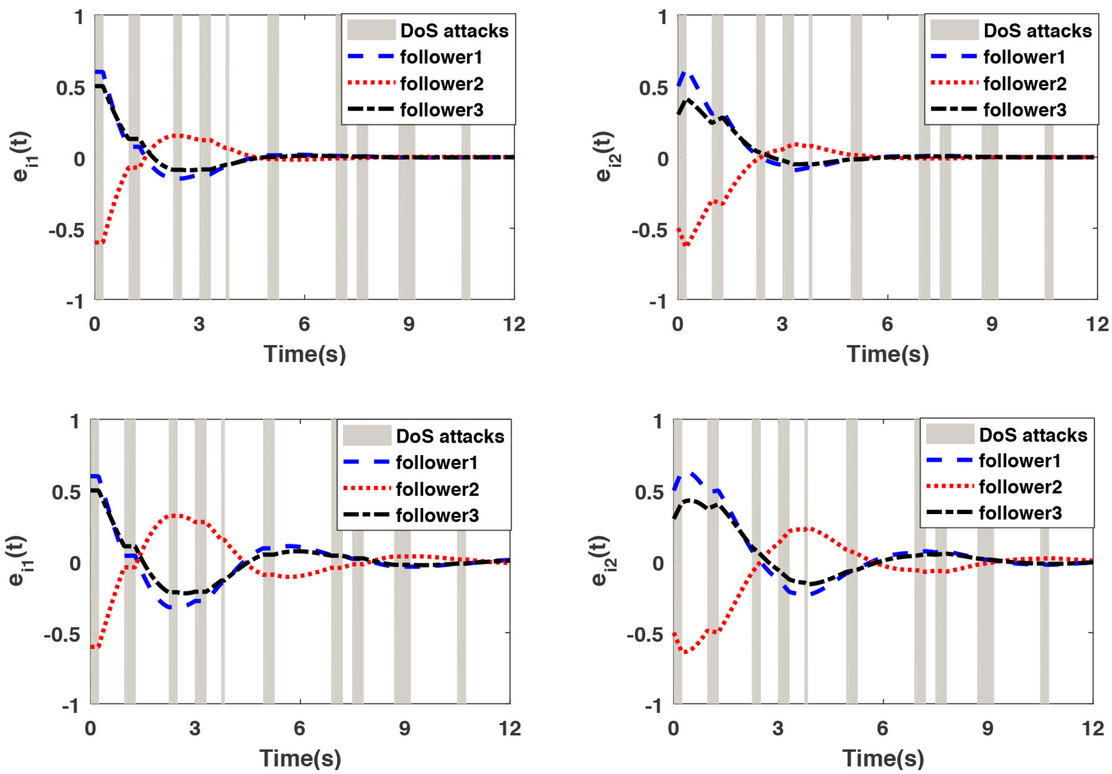

Fig. 7. Estimation errors $e_{i1}(t)$ and $e_{i2}(t)$ using Corollary 1 (up) and the traditional method (down) under DoS attacks.

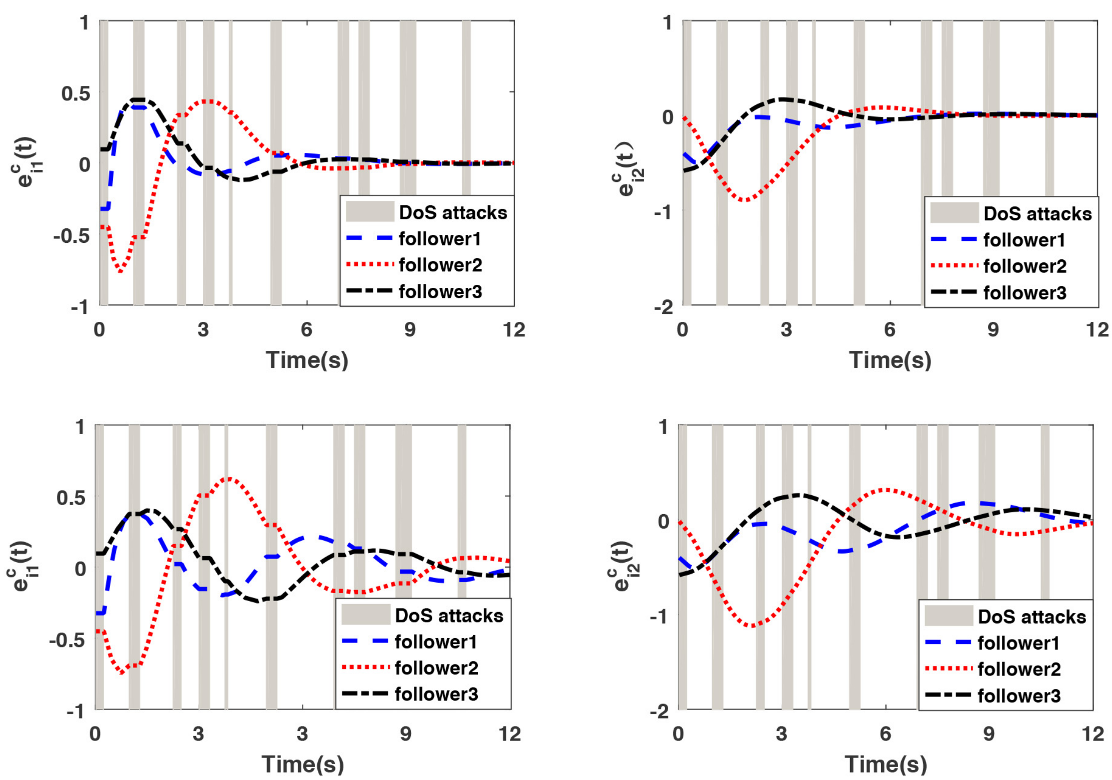

Fig. 8. Containment errors $e^c_{i1}(t)$ and $e^c_{i2}(t)$ using Corollary 1 (up) and the traditional method (down) under DoS attacks.

follower 2, and follower 3, respectively, which means that the communication rate is reduced by 65%, 67.25%, and 67%, respectively. Thus, the designed event-triggered mechanism can reduce effectively the number of controller updates.

VI. Conclusion

The problem of the observer-based event-triggered containment control for linear MASs in the case of DoS attacks is investigated in this article. An observer is used

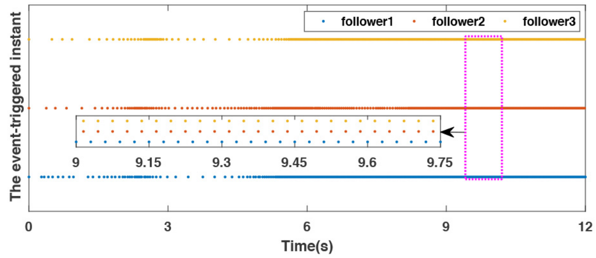

Fig. 9. Event-triggered instant for followers.

to estimate unmeasurable system states. Then, a distributed event-triggered containment controller using observer states is developed to achieve the states containment for MASs under DoS attacks, which can be easily converted into a convex problem by using an improved separation method. Besides, the Zeno behavior is also discussed and eliminated effectively by introducing a positive constant into the designed event-triggered mechanism. Simulation results show that the performance of UAVs using the proposed resilient controller is better than using the traditional controller. Finally, the observer-based resilient fully distributed containment control problem with a dynamic event-triggered mechanism for nonlinear MASs subjected to DoS attacks will be investigated in the future, which is inspired by [52].

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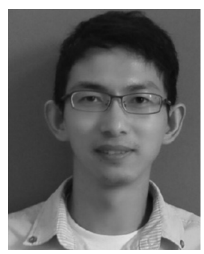

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